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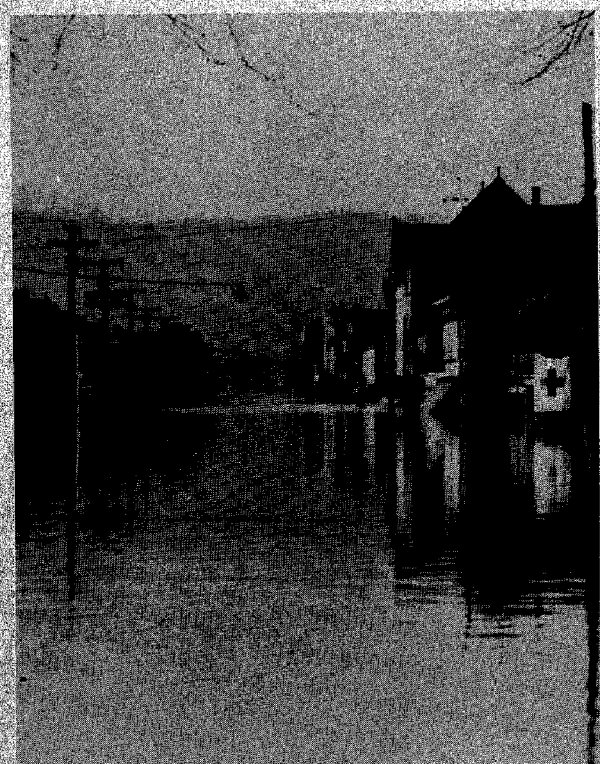
HYDROLOGY SUBCOMMITTEE

Guidelines For Determining
Flood Flow Frequency



Bulletin #17B
Revised September 1981
Editorial Corrections March 1982

INTERAGENCY ADVISORY COMMITTEE ON WATER DATA

U.S. Department of the Interior
Geological Survey
Office of Water Data Coordination

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of the
Hydrology Subcommittee

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Reston, Virginia 22092

FOREWORD

An accurate estimate of the flood damage potential is a key element to an effective, nationwide flood damage abatement program. Further, there is an acute need for a consistent approach to such estimates because management of the nation's water and related land resources is shared among various levels of government and private enterprise. To obtain both a consistent and accurate estimate of flood losses requires development, acceptance, and widespread application of a uniform, consistent and accurate technique for determining flood-flow frequencies.

In a pioneer attempt to promote a consistent approach to flood-flow frequency determination, the U.S. Water Resources Council in December 1967 published Bulletin No. 15, "A Uniform Technique for Determining Flood Flow Frequencies." The technique presented therein was adopted by the Council for use in all Federal planning involving water and related land resources. The Council also recommended use of the technique by State, local government, and private organizations. Adoption was based upon the clear understanding that efforts to develop methodological improvements in the technique would be continued and adopted when appropriate.

An extension and update of Bulletin No. 15 was published in March 1976 as Bulletin No. 17, "Guidelines for Determining Flood Flow Frequency." It presented the currently accepted methods for analyzing peak flow frequency data at gaging stations with sufficient detail to promote uniform application. The guide was a synthesis of studies undertaken to find methodological improvements and a survey of existing literature on peak flood flow determinations.

* The present guide is the second revision of the original publication *

and improves the methodologies. It revises and expands some of the techniques in the previous editions of this Bulletin and offers a further explanation of other techniques. It is the result of a continuing effort to develop a coherent set of procedures for accurately defining flood potentials. Much additional study is required before the two goals of accuracy and consistency will be fully attained. All who are interested in improving peak flood-flow frequency determinations are encouraged to submit comments, criticism and proposals to the Office of Water Data Coordination for consideration by the Hydrology Subcommittee.

Federal agencies are requested to use these guidelines in all planning activities involving water and related land resources. State, local and private organizations are encouraged to use these guidelines also to assure more uniformity, compatibility, and comparability in the frequency values that all concerned agencies and citizens must use for many vital decisions.

This present revision is adopted with the knowledge and understanding that review of these procedures will continue. When warranted by experience and by examination and testing of new techniques, other revisions will be published.

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The following pages contain revisions from material presented in Bulletin 17, "Guidelines for Determining Flood Flow Frequency."

1, 4, 8-2, and 13-1

The revised material is included on the lines enclosed by the + symbol.

The following pages of Bulletin 17 have been deleted:

13-2 through 13-35

The following pages contain revisions from the material in either Bulletin 17 or 17A.

i, ii, iii, iv, v, vi, vii, 1, 3, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 26, 1-1, 1-2, 1-3, 1-4, 2-3, 2-7, 2-8, 4-1, 5-1, 5-2, 5-3, 5-4, 6-1, 6-2, 6-3, 6-5, 6-6, 6-7, 7-1, 7-2, 7-3, 7-4, 7-5, 7-6, 7-7, 7-8, 7-9, 9-1 through 9-10, 10-1, 10-2, 10-3, 12-2 through 12-37 and 14-1

The revised material is included on the lines enclosed by the * symbol.

The following page of Bulletin 17 and 17A has been deleted from 17B: 4-2

Editorial corrections to Bulletin 17B were incorporated into this report in March 1982.

CONTENTS

I. Introduction

In December 1967, Bulletin No. 15, "A Uniform Technique for Determining Flood Flow Frequencies," was issued by the Hydrology Committee of the Water Resources Council. The report recommended use of the Pearson Type III distribution with log transformation of the data (log-Pearson Type III distribution) as a base method for flood flow frequency studies. As pointed out in that report, further studies were needed covering various aspects of flow frequency determinations.

+ In March 1976, Bulletin 17, "Guidelines for Determining Flood Flow Frequency" was issued by the Water Resources Council. The guide was an extension and update of Bulletin No. 15. It provided a more complete guide for flood flow frequency analysis incorporating currently accepted technical methods with sufficient detail to promote uniform application. It was limited to defining flood potentials in terms of peak discharge and exceedance probability at locations where a systematic record of peak flood flows is available. The recommended set of procedures was selected from those used or described in the literature prior to 1976, based on studies conducted for this purpose at the Center for Research in Water Resources of the University of Texas at Austin (summarized in Appendix 14) and on studies by the Work Group on Flood Flow Frequency. +

* The "Guidelines" were revised and reissued in June 1977 as Bulletin 17A. Bulletin 17B is the latest effort to improve and expand upon the earlier publications. Bulletin 17B provides revised procedures for weighting a station skew value with the results from a generalized skew study, detecting and treating outliers, making two station comparisons, and computing confidence limits about a frequency curve. The Work Group that prepared this revision did not address the suitability of the original distribution or the generalized skew map. *

Major problems are encountered when developing guides for flood flow frequency determinations. There is no procedure or set of procedures that can be adopted which, when rigidly applied to the available data, will accurately define the flood potential of any given watershed. Statistical analysis alone will not resolve all flood frequency problems. As discussed in subsequent sections of this guide, elements of risk and uncertainty are inherent in any flood frequency analysis. User decisions must be based on properly applied procedures and proper interpretation of results considering risk and uncertainty. Therefore, the judgment of a professional experienced in hydrologic analysis will enhance the usefulness of a flood frequency analysis and promote appropriate application.

It is possible to standarize many elements of flood frequency analysis. This guide describes each major element of the process of defining the flood potential at a specific location in terms of peak discharge and exceedance probability. Use is confined to stations where available records are adequate to warrant statistical analysis of the data. Special situations may require other approaches. In those cases where the procedures of this guide are not followed, deviations must be supported by appropriate study and accompanied by a comparison of results using the recommended procedures.

As a further means of achieving consistency and improving results, the Work Group recommends that studies be coordinated when more than one analyst is working currently on data for the same location. This recommendation holds particularly when defining exceedance probabilities for rare events, where this guide allows more latitude.

Flood records are limited. As more years of record become available at each location, the determination of flood potential may change. Thus, an estimate may be outdated a few years after it is made. Additional flood data alone may be sufficient reason for a fresh assessment of the flood potential. When making a new assessment, the analyst should incorporate in his study a review of earlier estimates. Where differences appear, they should be acknowledged and explained.

II. Summary

This guide describes the data and procedures for computing flood flow frequency curves where systematic stream gaging records of sufficient length (at least 10 years) to warrant statistical analysis are available as the basis for determination. The procedures do not cover watersheds where flood flows are appreciably altered by reservoir regulation or where the possibility of unusual events, such as dam failures, must be considered. The guide was specifically developed for the treatment of annual flood peak discharge. It is recognized that the same techniques could also be used to treat other hydrologic elements, such as flood volumes. Such applications, however, were not evaluated and are not intended.

The guide is divided into six broad sections which are summarized below:

A. Information to be Evaluated

The following categories of flood data are recognized: systematic records, historic data, comparison with similar watersheds, and flood estimates from precipitation. How each can be used to define the flood potential is briefly described.

B. Data Assumptions

A brief discussion of basic data assumptions is presented as a reminder to those developing flood flow frequency curves to be aware of potential data errors. Natural trends, randomness of events, watershed changes, mixed populations, and reliability of flow estimates are briefly discussed.

C. Determination of the Frequency Curve

This section provides the basic guide for determination of the frequency curve. The main thrust is determination of the annual flood series. Procedures are also recommended to convert an annual to partial-duration flood series.

The Pearson Type III distribution with log transformation of the flood data (log-Pearson Type III) is recommended as the basic distribution for defining the annual flood series. The method of moments is used to determine the statistical parameters of the distribution from station data. Generalized relations are used to modify the station skew coefficient. Methods are proposed for treatment of most flood record problems encountered. Procedures are described for refining the basic curve determined from statistical analysis of the systematic record and historic flood data to incorporate information gained from comparisons with similar watersheds and flood estimates from precipitation.

D. Reliability Applications

Procedures for computing confidence limits to the frequency curve are provided along with those for calculating risk and for making expected probability adjustments.

E. Potpourri

This section provides information of interest but not essential to the guide, including a discussion of non-conforming special situations, plotting positions, and suggested future studies.

F. Appendix

The appendix provides a list of references, a glossary and list of symbols, tables of K values, the computational details for treating most of the recommended procedures, information about how to obtain a computer program for handling the statistical analysis and treatment of data, and a summary of the report ("Flood Flow Frequency Techniques") describing studies made at the University of Texas which guided selection of some of the procedures proposed.

III. Information to be Evaluated

When developing a flood flow frequency curve, the analyst should consider all available information. The four general types of data which can be included in the flood flow frequency analysis are described in the following paragraphs. Specific applications are discussed in subsequent sections.

A. Systematic Records

Annual peak discharge information is observed systematically by many Federal and state agencies and private enterprises. Most annual peak records are obtained either from a continuous trace of river stages or from periodic observations of a crest-stage gage. Crest-stage records may provide information only on peaks above some preselected base. A major portion of these data are available in U.S. Geological Survey (USGS) Water Supply Papers and computer files, but additional information in published or unpublished form is available from other sources.

A statistical analysis of these data is the primary basis for the determination of the flow frequency curve for each station.

B. Historic Data

At many locations, particularly where man has occupied the flood plain for an extended period, there is information about major floods which occurred either before or after the period of systematic data collection. This information can often be used to make estimates of peak discharge. It also often defines an extended period during which the largest floods, either recorded or historic, are known. The USGS includes some historic flood information in its published reports and computer files. Additional information can sometimes be obtained from the files of other agencies or extracted from newspaper files or by intensive inquiry and investigation near the site for which the flood frequency information is needed.

Historic flood information should be obtained and documented whenever possible, particularly where the systematic record is relatively short. Use of historic data assures that estimates fit community experience and improves the frequency determinations.

C. Comparison With Similar Watersheds

Comparisons between computed frequency curves and maximum flood data of the watershed being investigated and those in a hydrologically similar region are useful for identification of unusual events and for testing the reasonableness of flood flow frequency determinations. Studies have been made and published [e.g., (1), (2), (3), (4)]* which permit comparing flood frequency estimates at a site with generalized estimates for a homogeneous region. Comparisons with information at stations in the immediate region should be made, particularly at gaging stations upstream and downstream, to promote regional consistency and help prevent gross errors.

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*Numbers in parentheses refer to numbered references in Appendix 1.

D. Flood Estimates From Precipitation

Flood discharges estimated from climatic data (rainfall and/or snowmelt) can be a useful adjunct to direct streamflow measurements. Such estimates, however, require at least adequate climatic data and a valid watershed model for converting precipitation to discharge. Unless such models are already calibrated to the watershed, considerable effort may be required to prepare such estimates.

Whether or not such studies are made will depend upon the availability of the information, the adequacy of the existing records, and the exceedance probability which is most important.

IV. Data Assumptions

Necessary assumptions for a statistical analysis are that the array of flood information is a reliable and representative time sample of random homogeneous events. Assessment of the adequacy and applicability of flood records is therefore a necessary first step in flood frequency analysis. This section discusses the effect of climatic trends, randomness of events, watershed changes, mixed populations, and reliability of flow estimates on flood frequency analysis.

A. Climatic Trends

There is much speculation about climatic changes. Available evidence indicates that major changes occur in time scales involving thousands of years. In hydrologic analysis it is conventional to assume flood flows are not affected by climatic trends or cycles. Climatic time invariance was assumed when developing this guide.

B. Randomness of Events

In general, an array of annual maximum peak flow rates may be considered a sample of random and independent events. Even when statistical tests of the serial correlation coefficients indicate a significant deviation from this assumption, the annual peak data may define an unbiased estimation of future flood activity if other assumptions are attained. The nonrandomness of the peak series will, however, increase the degree of uncertainty in the relation; that is, a relation based upon nonrandom data will have a degree of reliability attainable from a lesser sample of random data (5), (6).

C. Watershed Changes

It is becoming increasingly difficult to find watersheds in which the flow regime has not been altered by man's activity. Man's activities which can change flow conditions include urbanization, channelization, levees, the construction of reservoirs, diversions, and alteration of cover conditions.

Watershed history and flood records should be carefully examined to assure that no major watershed changes have occurred during the period of record. Documents which accompany flood records often list such changes. All watershed changes which affect record homogeneity, however, might not be listed; unlisted, for instance, might be the effects of urbanization and the construction of numerous small reservoirs over a period of several years. Such incremental changes may not significantly alter the flow regime from year to year but the cumulative effect can after several years.

Special effort should be made to identify those records which are not homogeneous. Only records which represent relatively constant watershed conditions should be used for frequency analysis.

D. Mixed Populations

At some locations flooding is created by different types of events. For example, flooding in some watersheds is created by snowmelt, rainstorms, or by combinations of both snowmelt and rainstorms. Such a record may not be homogeneous and may require special treatment.

E. Reliability of Flow Estimates

Errors exist in streamflow records, as in all other measured values. Errors in flow estimates are generally greatest during maximum flood flows. Measurement errors are usually random, and the variance introduced is usually small in comparison to the year-to-year variance in flood flows. The effects of measurement errors, therefore, may normally be neglected in flood flow frequency analysis. Peak flow estimates of historic floods can be substantially in error because of the uncertainty in both stage and stage-discharge relationships.

At times errors will be apparent or suspected. If substantial, the errors should be brought to the attention of the data collecting agency with supporting evidence and a request for a corrected value. A more complete discussion of sources of error in streamflow measurement is found in (7).

V. Determination of Frequency Curve

A. Series Selection

Flood events can be analyzed using either annual or partial-duration series. The annual flood series is based on the maximum flood peak for each year. A partial-duration series is obtained by taking all flood peaks equal to or greater than a predefined base flood.

If more than one flood per year must be considered, a partial-duration series may be appropriate. The base is selected to assure that all events of interest are evaluated including at least one event per time period. A major problem encountered when using a partial-duration series is to define flood events to ensure that all events are independent. It is common practice to establish an empirical basis for separating flood events. The basis for separation will depend upon the investigator and the intended use. No specific guidelines are recommended for defining flood events to be included in a partial series.

A study (8) was made to determine if a consistent relationship existed between the annual and partial series which could be used to convert from the annual to the partial-duration series. Based on this study as summarized in Appendix 14, the Work Group recommends that the partial-duration series be developed from observed data. An alternative but less desirable solution is to convert from the annual to the partial-duration series. For this, the first choice is to use a conversion factor specifically developed for the hydrologic region in which the gage is located. The second choice is to use published relationships [e.g., (9)].

Except for the preceding discussion of the the partial-duration series, the procedures described in this guide apply to the annual flood series.

B. Statistical Treatment

1. The Distribution--Flood events are a succession of natural events which, as far as can be determined, do not fit any one specific known statistical distribution. To make the problem of defining flood probabilities tractable it is necessary, however, to assign a distribution. Therefore, a study was sponsored to find which of many possible distributions and alternative fitting methods would best meet the purposes of this guide. This study is summarized in Appendix 14. The Work Group concluded from this and other studies that the Pearson Type III distribution with log transformation of the data (log-Pearson Type III distribution) should be the base method for analysis of annual series data using a generalized skew coefficient as described in the following section.

2. Fitting the Distribution--The recommended technique for fitting a log-Pearson Type III distribution to observed annual peaks is to compute the base 10 logarithms of the discharge, Q, at selected exceedance probability, P, by the equation:

$$\text{Log } Q=\overline{X}+KS \qquad (1)$$

where $\overline{X}$ and S are as defined below and K is a factor that is a function of the skew coefficient and selected exceedance probability. Values of K can be obtained from Appendix 3.

The mean, standard deviation and skew coefficient of station data may be computed using the following equations:

$$\overline{X} = \frac{\Sigma X}{N} \tag{2}$$

$$S = \left[\frac{\Sigma(X - \overline{X})^2}{(N - 1)}\right]^{0.5} \tag{3a}$$

$$= \left[\frac{(\Sigma X^2) - (\Sigma X)^2/N}{(N - 1)}\right]^{0.5} \tag{3b}$$

$$G = \frac{N \Sigma (X - \overline{X})^3}{(N - 1)(N - 2)S^3} \tag{4a}$$

$$= \frac{N^2(\Sigma X^3) - 3N(\Sigma X)(\Sigma X^2) + 2(\Sigma X)^3}{N(N-1)(N-2)S^3} \tag{4b}$$

in which:

X = logarithm of annual peak flow

N = number of items in data set

$\overline{X}$ = mean logarithm

S = standard deviation of logarithms

G = skew coefficient of logarithms

Formulas for computing the standard errors for the statistics $\overline{X}$, S, and G are given in Appendix 2. The precision of values computed with equations 3b and 4b is more sensitive than with equations 3a and 4a to the number of significant digits used in their calculation. When the available computation facilities only provide for a limited number of significant digits, equations 3a and 4a are preferable.

* 3. Estimating Generalized Skew--The skew coefficient of the station record (station skew) is sensitive to extreme events; thus it is difficult to obtain accurate skew estimates from small samples. The accuracy of the estimated skew coefficient can be improved by weighting the station skew with generalized skew estimated by pooling information from nearby sites. The following guidelines are recommended for estimating generalized skew.*

*Guidelines on weighting station and generalized skew are provided in the next section of this bulletin.

The recommended procedure for developing generalized skew coefficients requires the use of at least 40 stations, or all stations within a 100-mile radius. The stations used should have 25 or more years of record. It is recognized that in some locations a relaxation of these criteria may be necessary. The actual procedure includes analysis by three methods: 1) skew isolines drawn on a map; 2) skew prediction equation; and 3) the mean of the station skew values. Each of the methods are discussed separately.

To develop the isoline map, plot each station skew value at the centroid of its drainage basin and examine the plotted data for any geographic or topographic trends. If a pattern is evident, then isolines are drawn and the average of the squared differences between observed and isoline values, mean-square error (MSE), is computed. The MSE will be used in appraising the accuracy of the isoline map. If no pattern is evident, then an isoline map cannot be drawn and is therefore, not further considered.

A prediction equation should be developed that would relate either the station skew coefficients or the differences from the isoline map to predictor variables that affect the skew coefficient of the station record. These would include watershed and climatologic variables. The prediction equation should preferably be used for estimating the skew coefficient at stations with variables that are within the range of data used to calibrate the equation. The MSE (standard error of estimate squared) will be used to evaluate the accuracy of the prediction equation.

Determine the arithmetic mean and variance of the skew coefficients for all stations. In some cases the variability of the runoff regime may be so large as to preclude obtaining 40 stations with reasonably homogeneous hydrology. In these situations, the arithmetic mean and variance of about 20 stations may be used to estimate the generalized skew coefficient. The drainage areas and meteorologic, topographic, and geologic characteristics should be representative of the region around the station of interest.

Select the method that provides the most accurate skew coefficient *

* estimates. Compare the MSE from the isoline map to the MSE for the prediction equation. The smaller MSE should then be compared to the variance of the data. If the MSE is significantly smaller than the variance, the method with the smaller MSE should be used and that MSE used in equation 5 for $MSE_{\overline{G}}$. If the smaller MSE is not significantly smaller than the variance, neither the isoline map nor the prediction equation provides a more accurate estimate of the skew coefficient than does the mean value. The mean skew coefficient should be used as it provides the most accurate estimate and the variance should be used in equation 5 for $MSE_{\overline{G}}$.

In the absence of detailed studies the generalized skew ($\overline{G}$) can be read from Plate I found in the flyleaf pocket of this guide. This map of generalized skew was developed when this bulletin was first introduced and has not been changed. The procedures used to develop the statistical analysis for the individual stations do not conform in all aspects to the procedures recommended in the current guide. However, Plate I is still considered an alternative for use with the guide for those who prefer not to develop their own generalized skew procedures.

The accuracy of a regional generalized skew relationship is generally not comparable to Plate I accuracy. While the average accuracy of Plate I is given, the accuracy of subregions within the United States are not given. A comparison should only be made between relationships that cover approximately the same geographical area. Plate I accuracy would be directly comparable to other generalized skew relationships that are applicable to the entire country.

4. Weighting the Skew Coefficient--The station and generalized skew coefficient can be combined to form a better estimate of skew for a given watershed. Under the assumption that the generalized skew is unbiased and independent of station skew, the mean-square error (MSE) of the weighted estimate is minimized by weighting the station and generalized skew in inverse proportion to their individual mean-square errors. This concept is expressed in the following equation adopted from Tasker (39) which should be used in computing a weighted skew coefficient:

$$G_W = \frac{MSE_{\overline{G}}(G) + MSE_G(\overline{G})}{MSE_{\overline{G}} + MSE_G} \quad (5)$$

*

* where G_W = weighted skew coefficient

G = station skew

$\bar{G}$ = generalized skew

$MSE_{\bar{G}}$ = mean-square error of generalized skew

MSE_G = mean-square error of station skew

Equation 5 can be used to compute a weighted skew estimate regardless of the source of generalized skew, provided the MSE of the generalized skew can be estimated. When generalized skews are read from Plate I, the value of $MSE_{\bar{G}} = 0.302$ should be used in equation 5. The MSE of the station skew for log-Pearson Type III random variables can be obtained from the results of Monte Carlo experiments by Wallis, Matalas, and Slack (40). Their results show that the MSE of the logarithmic station skew is a function of record length and population skew. For use in calculating G_W, this function (MSE_G) can be approximated with sufficient accuracy by the equation:

$$MSE_G \approx 10^{[A - B\,[\mathrm{Log}_{10}(N/10)]]} \qquad (6)$$

Where
$$A = \begin{cases} -0.33 + 0.08|G| & \text{if } |G| \le 0.90 \\ -0.52 + 0.30|G| & \text{if } |G| > 0.90 \end{cases}$$

$$B = \begin{cases} 0.94 - 0.26|G| & \text{if } |G| \le 1.50 \\ 0.55 & \text{if } |G| > 1.50 \end{cases}$$

in which $|G|$ is the absolute value of the station skew (used as an estimate of population skew) and N is the record length in years. If the historic adjustment described in Appendix 6 has been applied, the historically adjusted skew, $\tilde{G}$, and historic period, H, are to be used for G and N, respectively, in equation 6. For convenience in manual computations, equation 6 was used to produce table 1 which shows MSE_G values for selected record lengths and station skews.

*

TABLE 1. - SUMMARY OF MEAN SQUARE ERROR OF STATION SKEW AS A FUNCTION OF RECORD LENGTH AND STATION SKEW. *

STATION SKEW (G OR $\tilde{G}$)	RECORD LENGTH, IN YEARS (N OR H) 10	20	30	40	50	60	70	80	90	100
0.0	0.468	0.244	0.167	0.127	0.103	0.087	0.075	0.066	0.059	0.054
0.1	0.476	0.253	0.175	0.134	0.109	0.093	0.080	0.071	0.064	0.058
0.2	0.485	0.262	0.183	0.142	0.116	0.099	0.086	0.077	0.069	0.063
0.3	0.494	0.272	0.192	0.150	0.123	0.105	0.092	0.082	0.074	0.068
0.4	0.504	0.282	0.201	0.158	0.131	0.113	0.099	0.089	0.080	0.073
0.5	0.513	0.293	0.211	0.167	0.139	0.120	0.106	0.095	0.087	0.079
0.6	0.522	0.303	0.221	0.176	0.148	0.128	0.114	0.102	0.093	0.086
0.7	0.532	0.315	0.231	0.186	0.157	0.137	0.122	0.110	0.101	0.093
0.8	0.542	0.326	0.243	0.196	0.167	0.146	0.130	0.118	0.109	0.100
0.9	0.562	0.345	0.259	0.211	0.181	0.159	0.142	0.130	0.119	0.111
1.0	0.603	0.376	0.285	0.235	0.202	0.178	0.160	0.147	0.135	0.126
1.1	0.646	0.410	0.315	0.261	0.225	0.200	0.181	0.166	0.153	0.143
1.2	0.692	0.448	0.347	0.290	0.252	0.225	0.204	0.187	0.174	0.163
1.3	0.741	0.488	0.383	0.322	0.281	0.252	0.230	0.212	0.197	0.185
1.4	0.794	0.533	0.422	0.357	0.314	0.283	0.259	0.240	0.224	0.211
1.5	0.851	0.581	0.465	0.397	0.351	0.318	0.292	0.271	0.254	0.240
1.6	0.912	0.623	0.498	0.425	0.376	0.340	0.313	0.291	0.272	0.257
1.7	0.977	0.667	0.534	0.456	0.403	0.365	0.335	0.311	0.292	0.275
1.8	1.047	0.715	0.572	0.489	0.432	0.391	0.359	0.334	0.313	0.295
1.9	1.122	0.766	0.613	0.523	0.463	0.419	0.385	0.358	0.335	0.316
2.0	1.202	0.821	0.657	0.561	0.496	0.449	0.412	0.383	0.359	0.339
2.1	1.288	0.880	0.704	0.601	0.532	0.481	0.442	0.410	0.385	0.363
2.2	1.380	0.943	0.754	0.644	0.570	0.515	0.473	0.440	0.412	0.389
2.3	1.479	1.010	0.808	0.690	0.610	0.552	0.507	0.471	0.442	0.417
2.4	1.585	1.083	0.866	0.739	0.654	0.592	0.543	0.505	0.473	0.447
2.5	1.698	1.160	0.928	0.792	0.701	0.634	0.582	0.541	0.507	0.479
2.6	1.820	1.243	0.994	0.849	0.751	0.679	0.624	0.580	0.543	0.513
2.7	1.950	1.332	1.066	0.910	0.805	0.728	0.669	0.621	0.582	0.550
2.8	2.089	1.427	1.142	0.975	0.862	0.780	0.716	0.666	0.624	0.589
2.9	2.239	1.529	1.223	1.044	0.924	0.836	0.768	0.713	0.669	0.631
3.0	2.399	1.638	1.311	1.119	0.990	0.895	0.823	0.764	0.716	0.676

*

* Application of equation 6 and table 1 to stations with absolute skew values (logs) greater than 2 and long periods of record gives relatively little weight to the station value. Application of equation 5 may also give improper weight to the generalized skew if the generalized and station skews differ by more than 0.5. In these situations, an examination of the data and the flood-producing characteristics of the watershed should be made and possibly greater weight given to the station skew. *

5. Broken Record--Annual peaks for certain years may be missing because of conditions not related to flood magnitude, such as gage removal. In this case, the different record segments are analyzed as a continuous record with length equal to the sum of both records, unless there is some physical change in the watershed between segments which may make the total record nonhomogeneous.

6. Incomplete Record--An incomplete record refers to a streamflow record in which some peak flows are missing because they were too low or too high to record, or the gage was out of operation for a short period because of flooding. Missing high and low data require different treatment.

When one or more high annual peaks during the period of systematic record have not been recorded, there is usually information available from which the peak discharge can be estimated. In most instances the data collecting agency routinely provides such estimates. If not, and such an estimate is made as part of the flood frequency analysis, it should be documented and the data collection agency advised.

At some crest gage sites the bottom of the gage is not reached *in some years. For this situation use of the conditional probability adjustment is recommended as described in Appendix 5. *

7. Zero Flood Years--Some streams in arid regions have no flow for the entire year. Thus, the annual flood series for these streams will have one or more zero flood values. This precludes the normal statistical analysis of the data using the recommended log-Pearson Type III *distribution because the logarithm of zero is minus infinity. The conditional probability adjustment is recommended for determining frequency curves for records with zero flood years as described in Appendix 5. *

8. Mixed Population--Flooding in some watersheds is created by different types of events. This results in flood frequency curves with abnormally large skew coefficients reflected by abnormal slope changes when plotted on logarithmic normal probability paper. In some situations the frequency curve of annual events can best be described by computing separate curves for each type of event. The curves are then combined.

Two examples of combinations of different types of flood-producing events include: (1) rain with snowmelt and (2) intense tropical storms with general cyclonic storms. Hydrologic factors and relationships operating during general winter rain flood are usually quite different from those operating during spring snowmelt floods or during local summer cloudburst floods. One example of mixed population is in the Sierra Nevada region of California. Frequency studies there have been made separately for rain floods which occur principally during the months of November through March, and for snowmelt floods, which occur during the months of April through July. Peak flows were segregated by cause--those predominately caused by snowmelt and those predominately caused by rain. Another example is along the Atlantic and Gulf Coasts, where in some instances floods from hurricane and nonhurricane events have been separated, thereby improving frequency estimates.

When it can be shown that there are two or more distinct and generally independent causes of floods it may be more reliable to segregate the flood data by cause, analyze each set separately, and then to combine the data sets using procedures such as described in (11). Separation by calendar periods in lieu of separation by events is not considered hydrologically reasonable unless the events in the separate periods are clearly caused by different hydrometeorologic conditions. The fitting procedures of this guide can be used to fit each flood series separately, with the exception that generalized skew coefficients cannot be used unless developed for specific type events being examined.

If the flood events that are believed to comprise two or more populations cannot be identified and separated by an objective and hydrologically meaningful criterion, the record shall be treated as coming from one population.

* 9. Outliers--Outliers are data points which depart significantly from the trend of the remaining data. The retention, modification, deletion of these outliers can significantly affect the statistical parameters computed from the data, especially for small samples. All procedures for treating outliers ultimately require judgment involving both mathematical and hydrologic considerations. The detection and treatment of high and low outliers are described below, and are outlined on the flow chart in Appendix 12 (figure 12-3).

If the station skew is greater than +0.4, tests for high outliers are considered first. If the station skew is less than -0.4 tests for low outliers are considered first. Where the station skew is between ± 0.4, tests for both high and low outliers should be applied before eliminating any outliers from the data set.

The following equation is used to detect high outliers:

$$X_H = \overline{X} + K_N S \tag{7}$$

where X_H = high outlier threshold in log units

$\overline{X}$ = mean logarithm of systematic peaks (X's) excluding zero flood events, peaks below gage base, and outliers previously detected.

S = standard deviation of X's

K_N = K value from Appendix 4 for sample size N

If the logarithms of peaks in a sample are greater than X_H in equation 7 then they are considered high outliers. Flood peaks considered high outliers should be compared with historic flood data and flood information at nearby sites. If information is available which indicated a high outlier(s) is the maximum in an extended period of time, the outlier(s) is treated as historic flood data as described in Section V.B.10. If useful **historic** information is not available to adjust for high outliers, then they should be retained as part of the systematic record. The treatment of all historic flood data and high outliers should be well documented in the analysis. *

* The following equation is used to detect low outliers:

$$X_L = \overline{X} - K_N S \qquad (8a)$$

where X_L = low outlier threshold in log units and the other terms are as defined for equation 7.

If an adjustment for historic flood data has previously been made, then the following equation is used to detect low outliers:

$$X_L = \tilde{M} - K_H \tilde{S} \qquad (8b)$$

where X_L = low outlier threshold in log units

K_H = K value from Appendix 4 for period used to compute $\tilde{M}$ and $\tilde{S}$

$\tilde{M}$ = historically adjusted mean logarithm

$\tilde{S}$ = historically adjusted standard deviation

If the logarithms of any annual peaks in a sample are less than X_L in equation 8a or b, then they are considered low outliers. Flood peaks considered low outliers are deleted from the record and the conditional probability adjustment described in Appendix 5 is applied.

If multiple values that have not been identified as outliers by the recommended procedure are very close to the threshold value, it may be desirable to test the sensitivity of the results to treating these values as outliers.

Use of the K values from Appendix 4 is equivalent to a one-sided test that detects outliers at the 10 percent level of significance (38). The K values are based on a normal distribution for detection of single outliers. In this Bulletin, the test is applied once and all values above the equation 7 threshold or below that from equation 8a or b are considered outliers. The selection of this outlier detection procedure was based on testing several procedures on simulated log-Pearson Type III and observed flood data and comparing results. The population skew coefficients for the simulated data were between $\pm$ 1.5, with skews for samples selected from these populations ranging between -3.67 and +3.25. The skew values * *for the observed data were between -2.19 and +2.80. Other test procedures evaluated included use of station, generalized, weighted, and zero skew. The selected procedure performed as well or better than the other procedures while at the same time being simple and easy to apply. Based on these results, this procedure is considered appropriate for use with the log-Pearson Type III distribution over the range of skews, $\pm$ 3.

10. Historic Flood Data - Information which indicates that any flood peaks which occurred before, during, or after the systematic record are maximums in an extended period of time should be used in frequency computations. Before such data are used, the reliability of the data, the peak discharge magnitude, changes in watershed conditions over the extended period of time, and the effects of these on the computed frequency curve must all be evaluated by the analyst. The adjustment described in Appendix 6 is recommended when historic data are used. The underlying assumption to this adjustment is that the data from the systematic record is representative of the intervening period between the systematic and historic record lengths. Comparison of results from systematic and historically adjusted analyses should be made.

The historic information should be used unless the comparison of the two analyses, the magnitude of the observed peaks, or other factors suggest that the historic data are not indicative of the extended record. All decisions made should be thoroughly documented. *

C. Refinements to Frequency Curve

The accuracy of flood probability estimates based upon statistical analysis of flood data deteriorates for probabilities more rare than those directly defined by the period of systematic record. This is partly because of the sampling error of the statistics from the station data and partly because the basic underlying distribution of flood data is not known exactly.

Although other procedures for estimating floods on a watershed and flood data from adjoining watersheds can sometimes be used for evaluating flood levels at high flows and rare exceedance probabilities;

procedures for doing so cannot be standardized to the same extent as the procedures discussed thus far. The purpose for which the flood frequency information is needed will determine the amount of time and effort that can justifiably be spent to obtain and make comparisons with other watersheds, and make and use flood estimates from precipitation. The remainder of the recommendations in this section are guides for use of these additional data to refine the flood frequency analysis.

The analyses to include when determining the flood magnitudes with 0.01 exceedance probability vary with length of systematic record as shown by an X in the following tabulation:

* Analyses to Include	Length of Record Available		
	10 to 24	25 to 50	50 or more
Statistical Analysis	X	X	X
Comparisons with Similar Watersheds	X	X	--
Flood Estimates from Precipitation	X	--	-- *

All types of analyses should be incorporated when defining flood magnitudes for exceedance probabilities of less than 0.01. The following sections explain how to include the various types of flood information in the analysis.

1. Comparisons with Similar Watersheds--A comparison between flood and storm records (see, e.g., (12)) and flood flow frequency analyses at nearby hydrologically similar watersheds will often aid in evaluating and interpreting both unusual flood experience and the flood frequency analysis of a given watershed. The shorter the flood record and the more unusual a given flood event, the greater will be the need for such comparisons.

Use of the weighted skew coefficient recommended by this guide is one form of regional comparison. Additional comparisons may be helpful and are described in the following paragraphs.

Several mathematical procedures have been proposed for adjusting a short record to reflect experience at a nearby long-term station. Such procedures usually yield useful results only when the gaging stations are on the same stream or in watersheds with centers not more than 50 miles apart. The recommended procedure for making such adjustments is given in Appendix 7. The use of such adjustments is confined to those situations where records are short and an improvement in accuracy of at least 10 percent can be demonstrated.

Comparisons and adjustment of a frequency curve based upon flood experience in nearby hydrologically similar watersheds can improve most flood frequency determinations. Comparisons of statistical parameters of the distribution of flows with selected exceedance probabilities can be made using prediction equations [e.g., (13), (14), (15), (16)], the index flood method (17), or simple drainage area plots. As these estimates are independent of the station analysis, a weighted average of the two estimates will be more accurate than either alone. The weight given to each estimate should be inversely proportional to its variance as described in Appendix 8. Recommendations of specific procedures for regional comparisons or for appraising the accuracy of such estimates are beyond the scope of this guide. In the absence of an accuracy appraisal, the accuracy of a regional estimate of a flood with 0.01 exceedance probability can be assumed equivalent to that from an analysis of a 10-year station record.

2. Flood Estimates from Precipitation--Floods estimated from observed or estimated precipitation (rainfall and/or snowmelt) can be used in several ways to improve definition of watershed flood potential. Such estimates, however, require a procedure (e.g., calibrated watershed model, unit hydrograph, rainfall-runoff relationships) for converting precipitation to discharge. Unless such procedures are available, considerable effort may be required to make these flood estimates. Whether or not such effort is warranted depends upon the procedures and data available and on the use to be made of the estimate.

Observed watershed precipitation can sometimes be used to estimate a missing maximum event in an incomplete flood record.

Observed watershed precipitation or precipitation observed at nearby stations in a meteorologically homogeneous region can be used to generate a synthetic record of floods for as many years as adequate precipitation records are available. Appraisal of the technique is outside the scope of this guide. Consequently, alternative procedures for making such studies, or criteria for deciding when available flood records should be extended by such procedures have not been evaluated.

Floods developed from precipitation estimates can be used to adjust frequency curves, including extrapolation beyond experienced values. Because of the many variables, no specific procedure is recommended at this time. Analysts making use of such procedures should first standardize methods for computing the flood to be used and then evaluate its probability of occurrence based upon flood and storm experience in a hydrologically and meteorologically homogeneous region. Plotting of the flood at the exceedance probability thus determined provides a guide for adjusting and extrapolating the frequency curve. Any adjustments must recognize the relative accuracy of the flood estimate and the other flood data.

VI. Reliability Application

The preceding sections have presented recommended procedures for determination of the flood frequency curve at a gaged location. When applying these curves to the solution of water resource problems, there are certain additional considerations which must be kept in mind. These are discussed in this section.

It is useful to make a distinction in hydrology between the concepts of risk and uncertainty (18).

Risk is a permanent population property of any random phenomenon such as floods. If the population distribution were known for floods, then the risk would be exactly known. The risk is stated as the probability that a specified flood magnitude will be exceeded in a specified period of years. Risk is inherent in the phenomenon itself and cannot be avoided.

Because use is made of data which are deficient, or biased, and because population properties must be estimated from these data by some technique, various errors and information losses are introduced into the flood frequency determination. Differences between the population properties and estimates of these properties derived from sample data constitute uncertainties. Risk can be decreased or minimized by various water resources developments and measures, while uncertainties can be decreased only by obtaining more or better data and by using better statistical techniques.

The following sections outline procedures to use for (a) computing confidence limits which can be used to evaluate the uncertainties inherent in the frequency determination, (b) calculating risk for specific time periods, and (c) adjusting the frequency curve to obtain the expected probability estimate. The recommendations given are guides as to how the procedures should be applied rather than instruction on when to apply them. Decisions on when to use each of the methods depend on the purpose of the estimate.

A. Confidence Limits

The user of frequency curves should be aware that the curve is only an estimate of the population curve; it is not an exact representation. A streamflow record is only a sample. How well this sample will predict the total flood experience (population) depends upon the sample size, its accuracy, and whether or not the underlying distribution is known. Confidence limits provide either a measure of the uncertainty of the estimated exceedance probability of a selected discharge or a measure of the uncertainty of the discharge at a selected exceedance probability. Confidence limits on the discharge can be computed by the procedure described in Appendix 9.

Application of confidence limits in reaching water resource planning decision depends upon the needs of the user. This discussion is presented to emphasize that the frequency curve developed using this guide is only today's best estimate of the flood frequency distribution. As more data become available, the estimate will normally be improved and the confidence limits narrowed.

B. Risk

As used in this guide, risk is defined as the probability that one or more events will exceed a given flood magnitude within a specified period of years. Accepting the flow frequency curve as accurately representing the flood exceedance probability, an estimate of risk may be computed for any selected time period. For a 1-year period the probability of exceedance, which is the reciprocal of the recurrence interval T, expresses the risk. Thus, there is a 1 percent chance that the 100-year flood will be exceeded in a given year. This statement however, ignores the considerable risk that a rare event will occur during the lifetime of a structure. The frequency curve can also be used to estimate the probability of a flood exceedance during a specified time period. For instance, there is a 50 percent chance that the flood with annual exceedance probability of 1 percent will be exceeded one or more times in the next 70 years.

Procedures for making these calculations are described in Appendix 10 and can be found in most standard hydrology texts or in (19) and (20).

C. Expected Probability

The expected probability is defined as the average of the true probabilities of all magnitude estimates for any specified flood frequency that might be made from successive samples of a specified size [(8), (21)]. It represents a measure of the central tendency of the spread between the confidence limits.

The study conducted for the Work Group (8) and summarized in Appendix 14 indicates that adjustments [(21),(22)] for the normal distribution are approximately correct for frequency curves computed using the statistical procedures described in this guide. Therefore, the committee recommends that if an expected probability adjustment is made, published adjustments applicable to the normal distribution be used. It would be the final step in the frequency analysis. It must be documented as to whether or not the expected probability adjustment is made. If curves are plotted, they must be appropriately labeled.

It should be recognized when using the expected probability adjustment that such adjustments are an attempt to incorporate the effects of uncertainty in application of the curve. The basic flood frequency curve without expected probability is the curve used in computation of confidence limits and risk and in obtaining weighted averages of independent estimates of flood frequency discharge.

The decision about use of the expected probability adjustment is a policy decision beyond the scope of this guide. It is most often used in estimates of annual flood damages and in establishing design flood criteria.

Appendix 11 provides precedures for computing the expected probability and further description of the concept.

VII. Potpourri

The following sections provide information that is of interest but not essential to use of this guide.

A. Non-conforming Special Situations

This guide describes the set of procedures recommended for defining flood potential as expressed by a flood flow frequency curve. In the Introduction the point is made that special situations may require other approaches and that in those cases where the procedures of this guide are not followed, deviations must be supported by appropriate study, including a comparison of the results obtained with those obtained using the recommended procedures.

It is not anticipated that many special situations warranting other approaches will occur. Detailed and specific recommendations on analysis are limited to the treatment of the station data including records of historic events. These procedures should be followed unless there are compelling technical reasons for departing from the guide procedures. These deviations are to be documented and supported by appropriate study, including comparison of results. The Hydrology Subcommittee asks that these situations be called to its attention for consideration in future modifications of this guide.

The map of skew (Plate I) is a generalized estimate. **Users are** encouraged to make detailed studies for their region of interest using the procedures outlined in Section V.B.3.

Major problems in flood frequency analysis at gaged locations are encountered when making flood estimates for probabilities more rare than defined by the available record. For these situations the guide described the information to incorporate in the analysis but allows considerable latitude in analysis.

B. Plotting Position

Calculations specified in this guide do not require designation of a plotting position. Section V.B.10., describing treatment of historic data, states that the results of the analysis should be shown graphically to permit an evaluation of the effect on the analysis of including historic data. The merits of alternative plotting position formulae were not studied and no recommendation is made.

A general formula for computing plotting positions (23) is

$$P = \frac{(m-a)}{(N-a-b+1)} \qquad (9)$$

where

m = the ordered sequence of flood values with the largest equal to 1

* N = number of items in data set and a and b depend upon the distribution. For symmetrical distributions a=b and the formula reduces to *

$$P = \frac{(m-a)}{(N-2a+1)} \qquad (10)$$

The Weibull plotting position in which a in equation 10 equals 0 was used to illustrate use of the historic adjustment of figure 6-3 and has been incorporated in the computer program referenced in Appendix 13, to facilitate data and analysis comparisons by the program user. This plotting position was used because it is analytically simple and intuitively easily understood (18, 24).

Weibull Plotting Position formula:

$$P = \frac{m}{N + 1} \tag{11}$$

C. Future Studies

This guide is designed to meet a current, ever-pressing demand that the Federal Government develop a coherent set of procedures for accurately defining flood potentials as needed in programs of flood damage abatement. Much additional study and data are required before the twin goals of accuracy and consistency will be obtained. It is hoped that this guide contributes to this effort by defining the essential elements of a coherent set of proedures for flood frequency determination. Although selection of the analytical procedures to be used in each step or element of the analysis has been carefully made based upon a review of the literature, the considerable practical experience of Work Group members, and special studies conducted to aid in the selection process, the need for additional studies is recognized. Following is a list of some additional needed studies identified by the Work Group.

1. Selection of distribution and fitting procedures
   - (a) Continued study of alternative distributions and fitting procedures is believed warranted.
   - (b) Initially the Work Group had expected to find that the proper distribution for a watershed would vary depending upon watershed and hydrometeorological conditions. Time did not permit exploration of this idea.

(c) More adequate criteria are needed for selection of a distribution.

(d) Development of techniques for evaluating homogeneity of series is needed.

2. The identification and treatment of mixed distributions.
3. The treatment of outliers both as to identification and computational procedures.
4. Alternative procedures for treating historic data.
5. More adequate computation procedures for confidence limits to the Pearson III distribution.
6. Procedures to incorporate flood estimates from precipitation into frequency analysis.
7. Guides for defining flood potentials for ungaged watersheds and watersheds with limited gaging records.
8. Guides for defining flood potentials for watersheds altered by urbanization and by reservoirs.

Appendix 1

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Appendix 2

GLOSSARY AND NOTATION

Glossary

The terms used in this guide include definitions taken from references listed in the Bibliography or from "Nomenclature for Hydraulics," Manual 43, American Society of Civil Engineers, 1962, and from definitions especially prepared for this guide. For more technical definitions of statistical terms, see "Dictionary of Statistical Terms" by M. G. Kendall and W. R. Buckland, Hafner Publishing Company, New York, 1957.

TERM	Definition
Annual Flood	The maximum momentary peak discharge in each year of record. (Sometimes the maximum mean daily discharge is used.)
Annual Flood Series	A list of annual floods.
Annual Series	A general term for a set of any kind of data in which each item is the maximum or minimum in a year.
Array	A list of data in order of magnitude; in flood-frequency analysis it is customary to list the largest value first, in a low-flow frequency analysis the smallest first.
Broken Record	A systematic record which is divided into separate continuous segments because of deliberate discontinuation of recording for significant periods of time.

Coefficient of Skewness — A numerical measure or index of the lack of symmetry in a frequency distribution. Function of the third moment of magnitudes about their mean, a measure of asymmetry. Also called "coefficient of skew" or "skew coefficient."

Confidence Limits — Computed values on both sides of an estimate of a parameter that show for a specified probability the range in which the true value of the parameter lies.

Distribution — Function describing the relative frequency with which events of various magnitudes occur.

Distribution-Free — Requiring no assumptions about the kind of probability distribution a set of data may have.

Exceedance Frequency — The percentage of values that exceed a specified magnitude, 100 times exceedance probability.

Exceedance Probability — Probability that a random event will exceed a specified magnitude in a given time period, usually one year unless otherwise indicated.

Expected Probability — The average of the true probabilities of all magnitude estimates for any specified flood frequency that might be made from successive samples of a specified size.

Generalized Skew Coefficient — A skew coefficient derived by a procedure which integrates values obtained at many locations.

Homogeneity — Records from the same populations.

Incomplete Record — A streamflow record in which some peak flows are missing because they were too low or high to record or the gage was out of operation for a short period because of flooding.

Level of Significance — The probability of rejecting a hypothesis when it is in fact true. At a "10-percent" level of significance the probability is 1/10.

* *Mean-Square Error* — Sum of the squared differences between the true and estimated values of a quantity divided by the number of observations. It can also be defined as the bias squared plus the variance of the quantity. *

$$\frac{\sum_{i=1}^{n} (X_{t_i} - X_{e_i})^2}{n}$$

Method of Moments — A standard statistical computation for estimating the moment of a distribution from the data of a sample.

Nonparametric — The same as distribution-free.

Normal Distribution — A probability distribution that is symmetrical about the mean, median, and mode (bell-shaped). It is the most studied distribution in statistics, even though most data are not exactly normally distributed, because of its value in theoretical work and because many other distributions can be transformed into normal. It is also known as Gaussian, The Laplacean, The Gauss-Laplace, or the Laplace-Gauss distribution, or the Second Law of Laplace.

Outlier — Outliers (extreme events) are data points which depart from the trend of the rest of data.

Parameter — A characteristic descriptor, such as a mean or standard deviation.

Percent Chance — A probability multiplied by 100.

Population — The entire (usually infinite) number of data from which a sample is taken or collected. The total number of past, present, and future floods at a location on a river is the population of floods for that location even if the floods are not measured or recorded.

Recurrence Interval (Return Period, Exceedance Interval) — The average time interval between actual occurrences of a hydrological event of a given or greater magnitude. In an annual flood series, the average interval in which a flood of a given size is exceeded as an annual maximum. In a partial duration series, the average interval between floods of a given size, regardless of their relationship to the year or any other period of time. The distinction holds even though for large floods recurrence intervals are nearly the same for both series.

Sample — An element, part, or fragment of a "population." Every hydrologic record is a sample of a much longer record.

Skew Coefficient — See "coefficient of skewness."

Standard Deviation	A measure of the dispersion or precision of a series of statistical values such as precipitation or streamflow. It is the square root of the sum of squares of the deviations from the arithmetic mean divided by the number of values or events in the series. It is now standard practice in statistics to divide by the number of values minus one in order to get an unbiased estimate of the variance from the sample data.
Standard Error	An estimate of the standard deviation of a statistic. Often calculated from a single set of observations. Calculated like the standard deviation but differing from it in meaning.
Student's t Distribution (t-distribution)	A distribution used in evaluation of variables which involve sample standard deviation rather than population standard deviation.
Test of Significance	A test made to learn the probability that a result is accidential or that a result differs from another result. For all the many types of tests there are standard formulas and tables. In making a test it is necessary to choose a "level of significance," the choice being arbitrary but generally not less than the low level of 10 percent nor more than the high level of 1 percent.

Transformation

The change of numerical values of data to make later computations easier, to linearize a plot or to normalize a skewed distribution by making it more nearly a normal distribution. The most common transformations are those changing ordinary numerical values into their logarithms, square roots or cube roots; many others are possible.

Variance

A measure of the amount of spread or dispersion of a set of values around their mean, obtained by calculating the mean value of the squares of the deviations from the mean, and hence equal to the square of the standard deviation.

Weighted Means

A value obtained by multiplying each of a series of values by its assigned weight and dividing the sum of those products by the sum of the weights.

Notation

Appendix notation is described in each Appendix. While most notation is consistent, slight variations do occur.

Notation	Explanation
* A	Fitting parameter used in equation 6. *
a	Variate in equations 9 and 10 which depends upon the distribution (23).
* B	Fitting parameter used in equation 6. *
b	Variate in equation 9 which depends upon the distribution (23)
G	Skew coefficient of logarithms of annual peak discharges
$\bar{G}$	Generalized skew coefficient
* $\tilde{G}$	Historically adjusted skew coefficient
G_W	Weighted skew coefficient
H	Historic record length
K_H	K value from Appendix 4 for historic period H *
K	Pearson Type III deviate
* K_N	K value from Appendix 4 for sample size N
$\tilde{M}$	Historically adjusted mean logarithm
MSE	Mean-square error
$MSE_{\bar{G}}$	Mean-square error of generalized skew *
MSE_G	Mean-square error of station skew
m	Ordered sequence of flood values, with the largest equal to 1
N	Number of items in data set
P	Exceedance probability
Q	Peak discharge, cfs
S	Standard deviation of logarithms of annual peak discharges
* $\tilde{S}$	Historically adjusted standard deviation *

Notation

SE_G Standard error of sample skew coefficient, which for samples from a normal distribution can be estimated as:

$$SE_G = \sqrt{\frac{6N(N-1)}{(N-2)(N+1)(N+3)}}$$

SE_S Standard error of sample standard deviation, can be estimated as:

$$SE_S = \frac{S\sqrt{1 + 0.75\ G^2}}{\sqrt{2N}}$$

$SE_{\overline{X}}$ Standard error of sample mean, can be estimated as:

$$SE_{\overline{X}} = \frac{S}{\sqrt{N}}$$

T Recurrence interval in years

X Logarithm of peak flow

$\overline{X}$ Mean logarithm of peak flows

* X_H High outlier threshold in log units *

X_L Low outlier threshold in log units

Appendix 3

TABLES OF K VALUES

The following table[1] contains K values for use in equation (1), for skew coefficients, G, from 0 to 9.0 and 0 to -9.0 and exceedance probabilities, P, from 0.9999 to 0.0001.

Approximate values of K can be obtained from the following transformation (26) when skew coefficients are between 1.0 and -1.0:

$$K = \frac{2}{G}\left[\left[\left(K_n - \frac{G}{6}\right)\frac{G}{6} + 1\right]^3 - 1\right] \qquad (3\text{-}1)$$

where K_n is the standard normal deviate and G is the skew coefficient. Because of the limitations (27) involved in use of this and other transforms, use of the table is preferred.

[1]This table was computed by Dr. H. Leon Harter and published in Technometrics, Vol. 11, No. 1, Feb. 1969, pp. 177-187, and Vol. 13, No. 1, Feb. 1971, pp. 203-204, "A New Table of Percentage Points of the Pearson Type III Distribution" and "More Percentage Points of the Pearson Distribution," respectively. These publications describe values only for positive coefficient of skew. Values for negative coefficient of skew were obtained by inverting the positive table and changing signs. The latter work was performed by the Central Technical Unit, SCS, Hyattsville, Md.

P	G =0.0	G =0.1	G =0.2	G =0.3	G =0.4	G =0.5	G =0.6
0.9999	-3.71902	-3.50703	-3.29921	-3.09631	-2.89907	-2.70836	-2.52507
0.9995	-3.29053	-3.12767	-2.96698	-2.80889	-2.65390	-2.50257	-2.35549
0.9990	-3.09023	-2.94834	-2.80786	-2.66915	-2.53261	-2.39867	-2.26780
0.9980	-2.87816	-2.75706	-2.63672	-2.51741	-2.39942	-2.28311	-2.16884
0.9950	-2.57583	-2.48187	-2.38795	-2.29423	-2.20092	-2.10825	-2.01644
0.9900	-2.32635	-2.25258	-2.17840	-2.10394	-2.02933	-1.95472	-1.88029
0.9800	-2.05375	-1.99973	-1.94499	-1.88959	-1.83361	-1.77716	-1.72033
0.9750	-1.95996	-1.91219	-1.86360	-1.81427	-1.76427	-1.71366	-1.66253
0.9600	-1.75069	-1.71580	-1.67999	-1.64329	-1.60574	-1.56740	-1.52830
0.9500	-1.64485	-1.61594	-1.58607	-1.55527	-1.52357	-1.49101	-1.45762
0.9000	-1.28155	-1.27037	-1.25824	-1.24516	-1.23114	-1.21618	-1.20028
0.8000	-0.84162	-0.84611	-0.84986	-0.85285	-0.85508	-0.85653	-0.85718
0.7000	-0.52440	-0.53624	-0.54757	-0.55839	-0.56867	-0.57840	-0.58757
0.6000	-0.25335	-0.26882	-0.28403	-0.29897	-0.31362	-0.32796	-0.34198
0.5704	-0.17733	-0.19339	-0.20925	-0.22492	-0.24037	-0.25558	-0.27047
0.5000	0.0	-0.01662	-0.03325	-0.04993	-0.06651	-0.08302	-0.09945
0.4296	0.17733	0.16111	0.14472	0.12820	0.11154	0.09478	0.07791
0.4000	0.25335	0.23763	0.22168	0.20552	0.18916	0.17261	0.15589
0.3000	0.52440	0.51207	0.49927	0.48600	0.47228	0.45812	0.44352
0.2000	0.84162	0.83639	0.83044	0.82377	0.81638	0.80829	0.79950
0.1000	1.28155	1.29178	1.30105	1.30936	1.31671	1.32309	1.32850
0.0500	1.64485	1.67279	1.69971	1.72562	1.75048	1.77428	1.79701
0.0400	1.75069	1.78462	1.81756	1.84949	1.88039	1.91022	1.93896
0.0250	1.95996	2.00688	2.05290	2.09795	2.14202	2.18505	2.22702
0.0200	2.05375	2.10697	2.15935	2.21081	2.26133	2.31084	2.35931
0.0100	2.32635	2.39961	2.47226	2.54421	2.61539	2.68572	2.75514
0.0050	2.57583	2.66965	2.76321	2.85636	2.94900	3.04102	3.13232
0.0020	2.87816	2.99978	3.12169	3.24371	3.36566	3.48737	3.60872
0.0010	3.09023	3.23322	3.37703	3.52139	3.66608	3.81090	3.95567
0.0005	3.29053	3.45513	3.62113	3.78820	3.95605	4.12443	4.29311
0.0001	3.71902	3.93453	4.15301	4.37394	4.59687	4.82141	5.04718

P	G =0.7	G =0.8	G =0.9	G =1.0	G =1.1	G =1.2	G =1.3
0.9999	-2.35015	-2.18448	-2.02891	-1.88410	-1.75053	-1.62838	-1.51752
0.9995	-2.21328	-2.07661	-1.94611	-1.82241	-1.70603	-1.59738	-1.49673
0.9990	-2.14053	-2.01739	-1.89894	-1.78572	-1.67825	-1.57695	-1.48216
0.9980	-2.05701	-1.94806	-1.84244	-1.74062	-1.64305	-1.55016	-1.46232
0.9950	-1.92580	-1.83660	-1.74919	-1.66390	-1.58110	-1.50114	-1.42439
0.9900	-1.80621	-1.73271	-1.66001	-1.58838	-1.51808	-1.44942	-1.38267
0.9800	-1.66325	-1.60604	-1.54886	-1.49188	-1.43529	-1.37929	-1.32412
0.9750	-1.61099	-1.55914	-1.50712	-1.45507	-1.40314	-1.35153	-1.30042
0.9600	-1.48852	-1.44813	-1.40720	-1.36584	-1.32414	-1.28225	-1.24028
0.9500	-1.42345	-1.38855	-1.35299	-1.31684	-1.28019	-1.24313	-1.20578
0.9000	-1.18347	-1.16574	-1.14712	-1.12762	-1.10726	-1.08608	-1.06413
0.8000	-0.85703	-0.85607	-0.85426	-0.85161	-0.84809	-0.84369	-0.83841
0.7000	-0.59615	-0.60412	-0.61146	-0.61815	-0.62415	-0.62944	-0.63400
0.6000	-0.35565	-0.36889	-0.38186	-0.39434	-0.40638	-0.41794	-0.42899
0.5704	-0.28516	-0.29961	-0.31368	-0.32740	-0.34075	-0.35370	-0.36620
0.5000	-0.11578	-0.13199	-0.14807	-0.16397	-0.17968	-0.19517	-0.21040
0.4296	0.06097	0.04397	0.02693	0.00987	-0.00719	-0.02421	-0.04116
0.4000	0.13901	0.12199	0.10486	0.08763	0.07032	0.05297	0.03560
0.3000	0.42851	0.41309	0.39729	0.38111	0.36458	0.34772	0.33054
0.2000	0.79002	0.77986	0.76902	0.75752	0.74537	0.73257	0.71915
0.1000	1.33294	1.33640	1.33889	1.34039	1.34092	1.34047	1.33904
0.0500	1.81864	1.83916	1.85856	1.87683	1.89395	1.90992	1.92472
0.0400	1.96660	1.99311	2.01848	2.04269	2.06573	2.08758	2.10823
0.0250	2.26790	2.30764	2.34623	2.38364	2.41984	2.45482	2.48855
0.0200	2.40670	2.45298	2.49811	2.54206	2.58480	2.62631	2.66657
0.0100	2.82359	2.89101	2.95735	3.02256	3.08660	3.14944	3.21103
0.0050	3.22281	3.31243	3.40109	3.48874	3.57530	3.66073	3.74497
0.0020	3.72957	3.84981	3.96932	4.08802	4.20582	4.32263	4.43839
0.0010	4.10022	4.24439	4.38807	4.53112	4.67344	4.81492	4.95549
0.0005	4.46189	4.63057	4.79899	4.96701	5.13449	5.30130	5.46735
0.0001	5.27389	5.50124	5.72899	5.95691	6.18480	6.41249	6.63980

P	G =1.4	G =1.5	G =1.6	G =1.7	G =1.8	G =1.9	G =2.0
0.9999	-1.41753	-1.32774	-1.24728	-1.17520	-1.11054	-1.05239	-0.99990
0.9995	-1.40413	-1.31944	-1.24235	-1.17240	-1.10901	-1.05159	-0.99950
0.9990	-1.39408	-1.31275	-1.23805	-1.16974	-1.10743	-1.05068	-0.99900
0.9980	-1.37981	-1.30279	-1.23132	-1.16534	-1.10465	-1.04898	-0.99800
0.9950	-1.35114	-1.28167	-1.21618	-1.15477	-1.09749	-1.04427	-0.99499
0.9900	-1.31815	-1.25611	-1.19680	-1.14042	-1.08711	-1.03695	-0.98995
0.9800	-1.26999	-1.21716	-1.16584	-1.11628	-1.06864	-1.02311	-0.97980
0.9750	-1.25004	-1.20059	-1.15229	-1.10537	-1.06001	-1.01640	-0.97468
0.9600	-1.19842	-1.15682	-1.11566	-1.07513	-1.03543	-0.99672	-0.95918
0.9500	-1.16827	-1.13075	-1.09338	-1.05631	-1.01973	-0.98381	-0.94871
0.9000	-1.04144	-1.01810	-0.99418	-0.96977	-0.94496	-0.91988	-0.89464
0.8000	-0.83223	-0.82516	-0.81720	-0.80837	-0.79868	-0.78816	-0.77686
0.7000	-0.63779	-0.64080	-0.64300	-0.64436	-0.64488	-0.64453	-0.64333
0.6000	-0.43949	-0.44942	-0.45873	-0.46739	-0.47538	-0.48265	-0.48917
0.5704	-0.37824	-0.38977	-0.40075	-0.41116	-0.42095	-0.43008	-0.43854
0.5000	-0.22535	-0.23996	-0.25422	-0.26808	-0.28150	-0.29443	-0.30685
0.4296	-0.05803	-0.07476	-0.09132	-0.10769	-0.12381	-0.13964	-0.15516
0.4000	0.01824	0.00092	-0.01631	-0.03344	-0.05040	-0.06718	-0.08371
0.3000	0.31307	0.29535	0.27740	0.25925	0.24094	0.22250	0.20397
0.2000	0.70512	0.69050	0.67532	0.65959	0.64335	0.62662	0.60944
0.1000	1.33665	1.33330	1.32900	1.32376	1.31760	1.31054	1.30259
0.0500	1.93836	1.95083	1.96213	1.97227	1.98124	1.98906	1.99573
0.0400	2.12768	2.14591	2.16293	2.17873	2.19332	2.20670	2.21888
0.0250	2.52102	2.55222	2.58214	2.61076	2.63810	2.66413	2.68888
0.0200	2.70556	2.74325	2.77964	2.81472	2.84848	2.88091	2.91202
0.0100	3.27134	3.33035	3.38804	3.44438	3.49935	3.55295	3.60517
0.0050	3.82798	3.90973	3.99016	4.06926	4.14700	4.22336	4.29832
0.0020	4.55304	4.66651	4.77875	4.88971	4.99937	5.10768	5.21461
0.0010	5.09505	5.23353	5.37087	5.50701	5.64190	5.77549	5.90776
0.0005	5.63252	5.79673	5.95990	6.12196	6.28285	6.44251	6.60090
0.0001	6.86661	7.09277	7.31818	7.54272	7.76632	7.98888	8.21034

P	G =2.1	G =2.2	G =2.3	G =2.4	G =2.5	G =2.6	G =2.7
0.9999	-0.95234	-0.90908	-0.86956	-0.83333	-0.80000	-0.76923	-0.74074
0.9995	-0.95215	-0.90899	-0.86952	-0.83331	-0.79999	-0.76923	-0.74074
0.9990	-0.95188	-0.90885	-0.86945	-0.83328	-0.79998	-0.76922	-0.74074
0.9980	-0.95131	-0.90854	-0.86929	-0.83320	-0.79994	-0.76920	-0.74073
0.9950	-0.94945	-0.90742	-0.86863	-0.83283	-0.79973	-0.76909	-0.74067
0.9900	-0.94607	-0.90521	-0.86723	-0.83196	-0.79921	-0.76878	-0.74049
0.9800	-0.93878	-0.90009	-0.86371	-0.82959	-0.79765	-0.76779	-0.73987
0.9750	-0.93495	-0.89728	-0.86169	-0.82817	-0.79667	-0.76712	-0.73943
0.9600	-0.92295	-0.88814	-0.85486	-0.82315	-0.79306	-0.76456	-0.73765
0.9500	-0.91458	-0.88156	-0.84976	-0.81927	-0.79015	-0.76242	-0.73610
0.9000	-0.86938	-0.84422	-0.81929	-0.79472	-0.77062	-0.74709	-0.72422
0.8000	-0.76482	-0.75211	-0.73880	-0.72495	-0.71067	-0.69602	-0.68111
0.7000	-0.64125	-0.63833	-0.63456	-0.62999	-0.62463	-0.61854	-0.61176
0.6000	-0.49494	-0.49991	-0.50409	-0.50744	-0.50999	-0.51171	-0.51263
0.5704	-0.44628	-0.45329	-0.45953	-0.46499	-0.46966	-0.47353	-0.47660
0.5000	-0.31872	-0.32999	-0.34063	-0.35062	-0.35992	-0.36852	-0.37640
0.4296	-0.17030	-0.18504	-0.19933	-0.21313	-0.22642	-0.23915	-0.25129
0.4000	-0.09997	-0.11590	-0.13148	-0.14665	-0.16138	-0.17564	-0.18939
0.3000	0.18540	0.16682	0.14827	0.12979	0.11143	0.09323	0.07523
0.2000	0.59183	0.57383	0.55549	0.53683	0.51789	0.49872	0.47934
0.1000	1.29377	1.28412	1.27365	1.26240	1.25039	1.23766	1.22422
0.0500	2.00128	2.00570	2.00903	2.01128	2.01247	2.01263	2.01177
0.0400	2.22986	2.23967	2.24831	2.25581	2.26217	2.26743	2.27160
0.0250	2.71234	2.73451	2.75541	2.77506	2.79345	2.81062	2.82658
0.0200	2.94181	2.97028	2.99744	3.02330	3.04787	3.07116	3.09320
0.0100	3.65600	3.70543	3.75347	3.80013	3.84540	3.88930	3.93183
0.0050	4.37186	4.44398	4.51467	4.58393	4.65176	4.71815	4.78313
0.0020	5.32014	5.42426	5.52694	5.62818	5.72796	5.82629	5.92316
0.0010	6.03865	6.16816	6.29626	6.42292	6.54814	6.67191	6.79421
0.0005	6.75798	6.91370	7.06804	7.22098	7.37250	7.52258	7.67121
0.0001	8.43064	8.64971	8.86753	9.08403	9.29920	9.51301	9.72543

P	G =2.8	G =2.9	G =3.0	G =3.1	G =3.2	G =3.3	G =3.4
0.9999	-0.71429	-0.68966	-0.66667	-0.64516	-0.62500	-0.60606	-0.58824
0.9995	-0.71429	-0.68966	-0.66667	-0.64516	-0.62500	-0.60606	-0.58824
0.9990	-0.71428	-0.68965	-0.66667	-0.64516	-0.62500	-0.60606	-0.58824
0.9980	-0.71428	-0.68965	-0.66667	-0.64516	-0.62500	-0.60606	-0.58824
0.9950	-0.71425	-0.68964	-0.66666	-0.64516	-0.62500	-0.60606	-0.58824
0.9900	-0.71415	-0.68959	-0.66663	-0.64514	-0.62499	-0.60606	-0.58823
0.9800	-0.71377	-0.68935	-0.66649	-0.64507	-0.62495	-0.60603	-0.58822
0.9750	-0.71348	-0.68917	-0.66638	-0.64500	-0.62491	-0.60601	-0.58821
0.9600	-0.71227	-0.68836	-0.66585	-0.64465	-0.62469	-0.60587	-0.58812
0.9500	-0.71116	-0.68759	-0.66532	-0.64429	-0.62445	-0.60572	-0.58802
0.9000	-0.70209	-0.68075	-0.66023	-0.64056	-0.62175	-0.60379	-0.58666
0.8000	-0.66603	-0.65086	-0.63569	-0.62060	-0.60567	-0.59096	-0.57652
0.7000	-0.60434	-0.59634	-0.58783	-0.57887	-0.56953	-0.55989	-0.55000
0.6000	-0.51276	-0.51212	-0.51073	-0.50863	-0.50585	-0.50244	-0.49844
0.5704	-0.47888	-0.48037	-0.48109	-0.48107	-0.48033	-0.47890	-0.47682
0.5000	-0.38353	-0.38991	-0.39554	-0.40041	-0.40454	-0.40792	-0.41058
0.4296	-0.26282	-0.27372	-0.28395	-0.29351	-0.30238	-0.31055	-0.31802
0.4000	-0.20259	-0.21523	-0.22726	-0.23868	-0.24946	-0.25958	-0.26904
0.3000	0.05746	0.03997	0.02279	0.00596	-0.01050	-0.02654	-0.04215
0.2000	0.45980	0.44015	0.42040	0.40061	0.38081	0.36104	0.34133
0.1000	1.21013	1.19539	1.18006	1.16416	1.14772	1.13078	1.11337
0.0500	2.00992	2.00710	2.00335	1.99869	1.99314	1.98674	1.97951
0.0400	2.27470	2.27676	2.27780	2.27785	2.27693	2.27506	2.27229
0.0250	2.84134	2.85492	2.86735	2.87865	2.88884	2.89795	2.90599
0.0200	3.11399	3.13356	3.15193	3.16911	3.18512	3.20000	3.21375
0.0100	3.97301	4.01286	4.05138	4.08859	4.12452	4.15917	4.19257
0.0050	4.84669	4.90884	4.96959	5.02897	5.08697	5.14362	5.19892
0.0020	6.01858	6.11254	6.20506	6.29613	6.38578	6.47401	6.56084
0.0010	6.91505	7.03443	7.15235	7.26881	7.38382	7.49739	7.60953
0.0005	7.81839	7.96411	8.10836	8.25115	8.39248	8.53236	8.67079
0.0001	9.93643	10.14602	10.35418	10.56090	10.76618	10.97001	11.17239

P	G =3.5	G =3.6	G =3.7	G =3.8	G =3.9	G =4.0	G =4.1
0.9999	-0.57143	-0.55556	-0.54054	-0.52632	-0.51282	-0.50000	-0.48780
0.9995	-0.57143	-0.55556	-0.54054	-0.52632	-0.51282	-0.50000	-0.48780
0.9990	-0.57143	-0.55556	-0.54054	-0.52632	-0.51282	-0.50000	-0.48780
0.9980	-0.57143	-0.55556	-0.54054	-0.52632	-0.51282	-0.50000	-0.48780
0.9950	-0.57143	-0.55556	-0.54054	-0.52632	-0.51282	-0.50000	-0.48780
0.9900	-0.57143	-0.55556	-0.54054	-0.52632	-0.51282	-0.50000	-0.48780
0.9800	-0.57142	-0.55555	-0.54054	-0.52631	-0.51282	-0.50000	-0.48780
0.9750	-0.57141	-0.55555	-0.54054	-0.52631	-0.51282	-0.50000	-0.48780
0.9600	-0.57136	-0.55552	-0.54052	-0.52630	-0.51281	-0.50000	-0.48780
0.9500	-0.57130	-0.55548	-0.54050	-0.52629	-0.51281	-0.49999	-0.48780
0.9000	-0.57035	-0.55483	-0.54006	-0.52600	-0.51261	-0.49986	-0.48772
0.8000	-0.56242	-0.54867	-0.53533	-0.52240	-0.50990	-0.49784	-0.48622
0.7000	-0.53993	-0.52975	-0.51952	-0.50929	-0.49911	-0.48902	-0.47906
0.6000	-0.49391	-0.48888	-0.48342	-0.47758	-0.47141	-0.46496	-0.45828
0.5704	-0.47413	-0.47088	-0.46711	-0.46286	-0.45819	-0.45314	-0.44777
0.5000	-0.41253	-0.41381	-0.41442	-0.41441	-0.41381	-0.41265	-0.41097
0.4296	-0.32479	-0.33085	-0.33623	-0.34092	-0.34494	-0.34831	-0.35105
0.4000	-0.27782	-0.28592	-0.29335	-0.30010	-0.30617	-0.31159	-0.31635
0.3000	-0.05730	-0.07195	-0.08610	-0.09972	-0.11279	-0.12530	-0.13725
0.2000	0.32171	0.30223	0.28290	0.26376	0.24484	0.22617	0.20777
0.1000	1.09552	1.07726	1.05863	1.03965	1.02036	1.00079	0.98096
0.0500	1.97147	1.96266	1.95311	1.94283	1.93186	1.92023	1.90796
0.0400	2.26862	2.26409	2.25872	2.25254	2.24558	2.23786	2.22940
0.0250	2.91299	2.91898	2.92397	2.92799	2.93107	2.93324	2.93450
0.0200	3.22641	3.23800	3.24853	3.25803	3.26653	3.27404	3.28060
0.0100	4.22473	4.25569	4.28545	4.31403	4.34147	4.36777	4.39296
0.0050	5.25291	5.30559	5.35698	5.40711	5.45598	5.50362	5.55005
0.0020	6.64627	6.73032	6.81301	6.89435	6.97435	7.05304	7.13043
0.0010	7.72024	7.82954	7.93744	8.04395	8.14910	8.25289	8.35534
0.0005	8.80779	8.94335	9.07750	9.21023	9.34158	9.47154	9.60013
0.0001	11.37334	11.57284	11.77092	11.96757	12.16280	12.35663	12.54906

P	G =4.2	G =4.3	G =4.4	G =4.5	G =4.6	G =4.7	G =4.8
0.9999	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9995	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9990	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9980	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9950	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9900	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9800	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9750	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9600	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9500	-0.47619	-0.46511	-0.45454	-0.44444	-0.43478	-0.42553	-0.41667
0.9000	-0.47614	-0.46508	-0.45452	-0.44443	-0.43477	-0.42553	-0.41666
0.8000	-0.47504	-0.46428	-0.45395	-0.44402	-0.43448	-0.42532	-0.41652
0.7000	-0.46927	-0.45967	-0.45029	-0.44114	-0.43223	-0.42357	-0.41517
0.6000	-0.45142	-0.44442	-0.43734	-0.43020	-0.42304	-0.41590	-0.40880
0.5704	-0.44212	-0.43623	-0.43016	-0.42394	-0.41761	-0.41121	-0.40477
0.5000	-0.40881	-0.40621	-0.40321	-0.39985	-0.39617	-0.39221	-0.38800
0.4296	-0.35318	-0.35473	-0.35572	-0.35619	-0.35616	-0.35567	-0.35475
0.4000	-0.32049	-0.32400	-0.32693	-0.32928	-0.33108	-0.33236	-0.33315
0.3000	-0.14861	-0.15939	-0.16958	-0.17918	-0.18819	-0.19661	-0.20446
0.2000	0.18967	0.17189	0.15445	0.13737	0.12067	0.10436	0.08847
0.1000	0.96090	0.94064	0.92022	0.89964	0.87895	0.85817	0.83731
0.0500	1.89508	1.88160	1.86757	1.85300	1.83792	1.82234	1.80631
0.0400	2.22024	2.21039	2.19988	2.18874	2.17699	2.16465	2.15174
0.0250	2.93489	2.93443	2.93314	2.93105	2.92818	2.92455	2.92017
0.0200	3.28622	3.29092	3.29473	3.29767	3.29976	3.30103	3.30149
0.0100	4.41706	4.44009	4.46207	4.48303	4.50297	4.52192	4.53990
0.0050	5.59528	5.63934	5.68224	5.72400	5.76464	5.80418	5.84265
0.0020	7.20654	7.28138	7.35497	7.42733	7.49847	7.56842	7.63718
0.0010	8.45646	8.55627	8.65479	8.75202	8.84800	8.94273	9.03623
0.0005	9.72737	9.85326	9.97784	10.10110	10.22307	10.34375	10.46318
0.0001	12.74010	12.92977	13.11808	13.30504	13.49066	13.67495	13.85794

P	G =4.9	G =5.0	G =5.1	G =5.2	G =5.3	G =5.4	G =5.5
0.9999	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9995	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9990	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9980	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9950	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9900	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9800	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9750	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9600	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9500	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9000	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.8000	-0.40806	-0.39993	-0.39211	-0.38458	-0.37734	-0.37036	-0.36363
0.7000	-0.40703	-0.39914	-0.39152	-0.38414	-0.37701	-0.37011	-0.36345
0.6000	-0.40177	-0.39482	-0.38799	-0.38127	-0.37469	-0.36825	-0.36196
0.5704	-0.39833	-0.39190	-0.38552	-0.37919	-0.37295	-0.36680	-0.36076
0.5000	-0.38359	-0.37901	-0.37428	-0.36945	-0.36453	-0.35956	-0.35456
0.4296	-0.35343	-0.35174	-0.34972	-0.34740	-0.34481	-0.34198	-0.33895
0.4000	-0.33347	-0.33336	-0.33284	-0.33194	-0.33070	-0.32914	-0.32729
0.3000	-0.21172	-0.21843	-0.22458	-0.23019	-0.23527	-0.23984	-0.24391
0.2000	0.07300	0.05798	0.04340	0.02927	0.01561	0.00243	-0.01028
0.1000	0.81641	0.79548	0.77455	0.75364	0.73277	0.71195	0.69122
0.0500	1.78982	1.77292	1.75563	1.73795	1.71992	1.70155	1.68287
0.0400	2.13829	2.12432	2.10985	2.09490	2.07950	2.06365	2.04739
0.0250	2.91508	2.90930	2.90283	2.89572	2.88796	2.87959	2.87062
0.0200	3.30116	3.30007	3.29823	3.29567	3.29240	3.28844	3.28381
0.0100	4.55694	4.57304	4.58823	4.60252	4.61594	4.62850	4.64022
0.0050	5.88004	5.91639	5.95171	5.98602	6.01934	6.05169	6.08307
0.0020	7.70479	7.77124	7.83657	7.90078	7.96390	8.02594	8.08691
0.0010	9.12852	9.21961	9.30952	9.39827	9.48586	9.57232	9.65766
0.0005	10.58135	10.69829	10.81401	10.92853	11.04186	11.15402	11.26502
0.0001	14.03963	14.22004	14.39918	14.57706	14.75370	14.92912	15.10332

P	G =5.6	G =5.7	G =5.8	G =5.9	G =6.0	G =6.1	G =6.2
0.9999	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9995	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9990	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9980	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9950	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9900	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9800	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9750	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9600	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9500	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9000	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.8000	-0.35714	-0.35087	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.7000	-0.35700	-0.35078	-0.34476	-0.33893	-0.33330	-0.32784	-0.32256
0.6000	-0.35583	-0.34985	-0.34402	-0.33836	-0.33285	-0.32750	-0.32230
0.5704	-0.35484	-0.34903	-0.34336	-0.33782	-0.33242	-0.32715	-0.32202
0.5000	-0.34955	-0.34455	-0.33957	-0.33463	-0.32974	-0.32492	-0.32016
0.4296	-0.33573	-0.33236	-0.32886	-0.32525	-0.32155	-0.31780	-0.31399
0.4000	-0.32519	-0.32285	-0.32031	-0.31759	-0.31472	-0.31171	-0.30859
0.3000	-0.24751	-0.25064	-0.25334	-0.25562	-0.25750	-0.25901	-0.26015
0.2000	-0.02252	-0.03427	-0.04553	-0.05632	-0.06662	-0.07645	-0.08580
0.1000	0.67058	0.65006	0.62966	0.60941	0.58933	0.56942	0.54970
0.0500	1.66390	1.64464	1.62513	1.60538	1.58541	1.56524	1.54487
0.0400	2.03073	2.01369	1.99629	1.97855	1.96048	1.94210	1.92343
0.0250	2.86107	2.85096	2.84030	2.82912	2.81743	2.80525	2.79259
0.0200	3.27854	3.27263	3.26610	3.25898	3.25128	3.24301	3.23419
0.0100	4.65111	4.66120	4.67050	4.67903	4.68680	4.69382	4.70013
0.0050	6.11351	6.14302	6.17162	6.19933	6.22616	6.25212	6.27723
0.0020	8.14683	8.20572	8.26359	8.32046	8.37634	8.43125	8.48519
0.0010	9.74190	9.82505	9.90713	9.98815	10.06812	10.14706	10.22499
0.0005	11.37487	11.48360	11.59122	11.69773	11.80316	11.90752	12.01082
0.0001	15.27632	15.44813	15.61878	15.78826	15.95660	16.12380	16.28989

P	G =6.3	G =6.4	G =6.5	G =6.6	G =6.7	G =6.8	G =6.9
0.9999	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9995	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9990	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9980	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9950	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9900	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9800	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9750	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9600	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9500	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9000	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.8000	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.7000	-0.31745	-0.31249	-0.30769	-0.30303	-0.29850	-0.29412	-0.28985
0.6000	-0.31724	-0.31234	-0.30757	-0.30294	-0.29844	-0.29407	-0.28982
0.5704	-0.31702	-0.31216	-0.30743	-0.30283	-0.29835	-0.29400	-0.28977
0.5000	-0.31549	-0.31090	-0.30639	-0.30198	-0.29766	-0.29344	-0.28931
0.4296	-0.31016	-0.30631	-0.30246	-0.29862	-0.29480	-0.29101	-0.28726
0.4000	-0.30538	-0.30209	-0.29875	-0.29537	-0.29196	-0.28854	-0.28511
0.3000	-0.26097	-0.26146	-0.26167	-0.26160	-0.26128	-0.26072	-0.25995
0.2000	-0.09469	-0.10311	-0.11107	-0.11859	-0.12566	-0.13231	-0.13853
0.1000	0.53019	0.51089	0.49182	0.47299	0.45440	0.43608	0.41803
0.0500	1.52434	1.50365	1.48281	1.46186	1.44079	1.41963	1.39839
0.0400	1.90449	1.88528	1.86584	1.84616	1.82627	1.80618	1.78591
0.0250	2.77947	2.76591	2.75191	2.73751	2.72270	2.70751	2.69195
0.0200	3.22484	3.21497	3.20460	3.19374	3.18241	3.17062	3.15838
0.0100	4.70571	4.71061	4.71482	4.71836	4.72125	4.72350	4.72512
0.0050	6.30151	6.32497	6.34762	6.36948	6.39055	6.41086	6.43042
0.0020	8.53820	8.59027	8.64142	8.69167	8.74102	8.78950	8.83711
0.0010	10.30192	10.37785	10.45281	10.52681	10.59986	10.67197	10.74316
0.0005	12.11307	12.21429	12.31450	12.41370	12.51190	12.60913	12.70539
0.0001	16.45487	16.61875	16.78156	16.94329	17.10397	17.26361	17.42221

P	G =7.0	G =7.1	G =7.2	G =7.3	G =7.4	G =7.5	G =7.6
0.9999	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9995	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9990	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9980	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9950	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9900	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9800	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9750	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9600	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9500	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9000	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.8000	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.7000	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.6000	-0.28569	-0.28167	-0.27776	-0.27396	-0.27026	-0.26666	-0.26315
0.5704	-0.28565	-0.28164	-0.27774	-0.27394	-0.27025	-0.26665	-0.26315
0.5000	-0.28528	-0.28135	-0.27751	-0.27376	-0.27010	-0.26654	-0.26306
0.4296	-0.28355	-0.27990	-0.27629	-0.27274	-0.26926	-0.26584	-0.26248
0.4000	-0.28169	-0.27829	-0.27491	-0.27156	-0.26825	-0.26497	-0.26175
0.3000	-0.25899	-0.25785	-0.25654	-0.25510	-0.25352	-0.25183	-0.25005
0.2000	-0.14434	-0.14975	-0.15478	-0.15942	-0.16371	-0.16764	-0.17123
0.1000	0.40026	0.38277	0.36557	0.34868	0.33209	0.31582	0.29986
0.0500	1.37708	1.35571	1.33430	1.31287	1.29141	1.26995	1.24850
0.0400	1.76547	1.74487	1.72412	1.70325	1.68225	1.66115	1.63995
0.0250	2.67603	2.65977	2.64317	2.62626	2.60905	2.59154	2.57375
0.0200	3.14572	3.13263	3.11914	3.10525	3.09099	3.07636	3.06137
0.0100	4.72613	4.72653	4.72635	4.72559	4.72427	4.72240	4.71998
0.0050	6.44924	6.46733	6.48470	6.50137	6.51735	6.53264	6.54727
0.0020	8.88387	8.92979	8.97488	9.01915	9.06261	9.10528	9.14717
0.0010	10.81343	10.88281	10.95129	11.01890	11.08565	11.15154	11.21658
0.0005	12.80069	12.89505	12.98848	13.08098	13.17258	13.26328	13.35309
0.0001	17.57979	17.73636	17.89193	18.04652	18.20013	18.35278	18.50447

P	G =7.7	G =7.8	G =7.9	G =8.0	G =8.1	G =8.2	G =8.3
0.9999	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9995	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9990	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9980	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9950	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9900	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9800	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9750	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9600	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9500	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9000	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.8000	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.7000	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.6000	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.5704	-0.25973	-0.25640	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.5000	-0.25966	-0.25635	-0.25312	-0.24996	-0.24689	-0.24388	-0.24095
0.4296	-0.25919	-0.25596	-0.25280	-0.24970	-0.24667	-0.24371	-0.24081
0.4000	-0.25857	-0.25544	-0.25236	-0.24933	-0.24637	-0.24345	-0.24060
0.3000	-0.24817	-0.24622	-0.24421	-0.24214	-0.24003	-0.23788	-0.23571
0.2000	-0.17450	-0.17746	-0.18012	-0.18249	-0.18459	-0.18643	-0.18803
0.1000	0.28422	0.26892	0.25394	0.23929	0.22498	0.21101	0.19737
0.0500	1.22706	1.20565	1.18427	1.16295	1.14168	1.12048	1.09936
0.0400	1.61867	1.59732	1.57591	1.55444	1.53294	1.51141	1.48985
0.0250	2.55569	2.53737	2.51881	2.50001	2.48099	2.46175	2.44231
0.0200	3.04604	3.03038	3.01439	2.99810	2.98150	2.96462	2.94746
0.0100	4.71704	4.71358	4.70961	4.70514	4.70019	4.69476	4.68887
0.0050	6.56124	6.57456	6.58725	6.59931	6.61075	6.62159	6.63183
0.0020	9.18828	9.22863	9.26823	9.30709	9.34521	9.38262	9.41931
0.0010	11.28080	11.34419	11.40677	11.46855	11.52953	11.58974	11.64917
0.0005	13.44202	13.53009	13.61730	13.70366	13.78919	13.87389	13.95778
0.0001	18.65522	18.80504	18.95393	19.10191	19.24898	19.39517	19.54046

P	G =8.4	G =8.5	G =8.6	G =8.7	G =8.8	G =8.9	G =9.0
0.9999	-0.23810	-0.23529	-0.23256	-0.22989	-0.22727	-0.22472	-0.22222
0.9995	-0.23810	-0.23529	-0.23256	-0.22989	-0.22727	-0.22472	-0.22222
0.9990	-0.23810	-0.23529	-0.23256	-0.22989	-0.22727	-0.22472	-0.22222
0.9980	-0.23810	-0.23529	-0.23256	-0.22989	-0.22727	-0.22472	-0.22222
0.9950	-0.23810	-0.23529	-0.23256	-0.22989	-0.22727	-0.22472	-0.22222
0.9900	-0.23810	-0.23529	-0.23256	-0.22989	-0.22727	-0.22472	-0.22222
0.9800	-0.23810	-0.23529	-0.23256	-0.22989	-0.22727	-0.22472	-0.22222
0.9750	-0.23810	-0.23529	-0.23256	-0.22989	-0.22727	-0.22472	-0.22222
0.9600	-0.23810	-0.23529	-0.23256	-0.22989	-0.22727	-0.22472	-0.22222
0.9500	-0.23810	-0.23529	-0.23256	-0.22989	-0.22727	-0.22472	-0.22222
0.9000	-0.23810	-0.23529	-0.23256	-0.22989	-0.22727	-0.22472	-0.22222
0.8000	-0.23810	-0.23529	-0.23256	-0.22989	-0.22727	-0.22472	-0.22222
0.7000	-0.23810	-0.23529	-0.23256	-0.22989	-0.22727	-0.22472	-0.22222
0.6000	-0.23810	-0.23529	-0.23256	-0.22988	-0.22727	-0.22472	-0.22222
0.5704	-0.23809	-0.23529	-0.23256	-0.22988	-0.22727	-0.22472	-0.22222
0.5000	-0.23808	-0.23528	-0.23255	-0.22988	-0.22727	-0.22472	-0.22222
0.4296	-0.23797	-0.23520	-0.23248	-0.22982	-0.22722	-0.22468	-0.22219
0.4000	-0.23779	-0.23505	-0.23236	-0.22972	-0.22714	-0.22461	-0.22214
0.3000	-0.23352	-0.23132	-0.22911	-0.22690	-0.22469	-0.22249	-0.22030
0.2000	-0.18939	-0.19054	-0.19147	-0.19221	-0.19277	-0.19316	-0.19338
0.1000	0.18408	0.17113	0.15851	0.14624	0.13431	0.12272	0.11146
0.0500	1.07832	1.05738	1.03654	1.01581	0.99519	0.97471	0.95435
0.0400	1.46829	1.44673	1.42518	1.40364	1.38213	1.36065	1.33922
0.0250	2.42268	2.40287	2.38288	2.36273	2.34242	2.32197	2.30138
0.0200	2.93002	2.91234	2.89440	2.87622	2.85782	2.83919	2.82035
0.0100	4.68252	4.67573	4.66850	4.66085	4.65277	4.64429	4.63541
0.0050	6.64148	6.65056	6.65907	6.66703	6.67443	6.68130	6.68763
0.0020	9.45530	9.49060	9.52521	9.55915	9.59243	9.62504	9.65701
0.0010	11.70785	11.76576	11.82294	11.87938	11.93509	11.99009	12.04437
0.0005	14.04086	14.12314	14.20463	14.28534	14.36528	14.44446	14.52288
0.0001	19.68489	19.82845	19.97115	20.11300	20.25402	20.39420	20.53356

P	G =-0.0	G =-0.1	G =-0.2	G =-0.3	G =-0.4	G =-0.5	G =-0.6
0.9999	-3.71902	-3.93453	-4.15301	-4.37394	-4.59687	-4.82141	-5.04718
0.9995	-3.29053	-3.45513	-3.62113	-3.78820	-3.95605	-4.12443	-4.29311
0.9990	-3.09023	-3.23322	-3.37703	-3.52139	-3.66608	-3.81090	-3.95567
0.9980	-2.87816	-2.99978	-3.12169	-3.24371	-3.36566	-3.48737	-3.60872
0.9950	-2.57583	-2.66965	-2.76321	-2.85636	-2.94900	-3.04102	-3.13232
0.9900	-2.32635	-2.39961	-2.47226	-2.54421	-2.61539	-2.68572	-2.75514
0.9800	-2.05375	-2.10697	-2.15935	-2.21081	-2.26133	-2.31084	-2.35931
0.9750	-1.95996	-2.00688	-2.05290	-2.09795	-2.14202	-2.18505	-2.22702
0.9600	-1.75069	-1.78462	-1.81756	-1.84949	-1.88039	-1.91022	-1.93896
0.9500	-1.64485	-1.67279	-1.69971	-1.72562	-1.75048	-1.77428	-1.79701
0.9000	-1.28155	-1.29178	-1.30105	-1.30936	-1.31671	-1.32309	-1.32850
0.8000	-0.84162	-0.83639	-0.83044	-0.82377	-0.81638	-0.80829	-0.79950
0.7000	-0.52440	-0.51207	-0.49927	-0.48600	-0.47228	-0.45812	-0.44352
0.6000	-0.25335	-0.23763	-0.22168	-0.20552	-0.18916	-0.17261	-0.15589
0.5704	-0.17733	-0.16111	-0.14472	-0.12820	-0.11154	-0.09478	-0.07791
0.5000	0.0	0.01662	0.03325	0.04993	0.06651	0.08302	0.09945
0.4296	0.17733	0.19339	0.20925	0.22492	0.24037	0.25558	0.27047
0.4000	0.25335	0.26882	0.28403	0.29897	0.31362	0.32796	0.34198
0.3000	0.52440	0.53624	0.54757	0.55839	0.56867	0.57840	0.58757
0.2000	0.84162	0.84611	0.84986	0.85285	0.85508	0.85653	0.85718
0.1000	1.28155	1.27037	1.25824	1.24516	1.23114	1.21618	1.20028
0.0500	1.64485	1.61594	1.58607	1.55527	1.52357	1.49101	1.45762
0.0400	1.75069	1.71580	1.67999	1.64329	1.60574	1.56740	1.52830
0.0250	1.95996	1.91219	1.86360	1.81427	1.76427	1.71366	1.66253
0.0200	2.05375	1.99973	1.94499	1.88959	1.83361	1.77716	1.72033
0.0100	2.32635	2.25258	2.17840	2.10394	2.02933	1.95472	1.88029
0.0050	2.57583	2.48187	2.38795	2.29423	2.20092	2.10825	2.01644
0.0020	2.87816	2.75706	2.63672	2.51741	2.39942	2.28311	2.16884
0.0010	3.09023	2.94834	2.80786	2.66915	2.53261	2.39867	2.26780
0.0005	3.29053	3.12767	2.96698	2.80889	2.65390	2.50257	2.35549
0.0001	3.71902	3.50703	3.29921	3.09631	2.89907	2.70836	2.52507

P	G1=-0.7	G1=-0.8	G1=-0.9	G1=-1.0	G1=-1.1	G1=-1.2	G1=-1.3
0.9999	-5.27389	-5.50124	-5.72899	-5.95691	-6.18480	-6.41249	-6.63980
0.9995	-4.46189	-4.63057	-4.79899	-4.96701	-5.13449	-5.30130	-5.46735
0.9990	-4.10022	-4.24439	-4.38807	-4.53112	-4.67344	-4.81492	-4.95549
0.9980	-3.72957	-3.84981	-3.96932	-4.08802	-4.20582	-4.32263	-4.43839
0.9950	-3.22281	-3.31243	-3.40109	-3.48874	-3.57530	-3.66073	-3.74497
0.9900	-2.82359	-2.89101	-2.95735	-3.02256	-3.08660	-3.14944	-3.21103
0.9800	-2.40670	-2.45298	-2.49811	-2.54206	-2.58480	-2.62631	-2.66657
0.9750	-2.26790	-2.30764	-2.34623	-2.38364	-2.41984	-2.45482	-2.48855
0.9600	-1.96660	-1.99311	-2.01848	-2.04269	-2.06573	-2.08758	-2.10823
0.9500	-1.81864	-1.83916	-1.85856	-1.87683	-1.89395	-1.90992	-1.92472
0.9000	-1.33294	-1.33640	-1.33889	-1.34039	-1.34092	-1.34047	-1.33904
0.8000	-0.79002	-0.77986	-0.76902	-0.75752	-0.74537	-0.73257	-0.71915
0.7000	-0.42851	-0.41309	-0.39729	-0.38111	-0.36458	-0.34772	-0.33054
0.6000	-0.13901	-0.12199	-0.10486	-0.08763	-0.07032	-0.05297	-0.03560
0.5704	-0.06097	-0.04397	-0.02693	-0.00987	0.00719	0.02421	0.04116
0.5000	0.11578	0.13199	0.14807	0.16397	0.17968	0.19517	0.21040
0.4296	0.28516	0.29961	0.31368	0.32740	0.34075	0.35370	0.36620
0.4000	0.35565	0.36889	0.38186	0.39434	0.40638	0.41794	0.42899
0.3000	0.59615	0.60412	0.61146	0.61815	0.62415	0.62944	0.63400
0.2000	0.85703	0.85607	0.85426	0.85161	0.84809	0.84369	0.83841
0.1000	1.18347	1.16574	1.14712	1.12762	1.10726	1.08608	1.06413
0.0500	1.42345	1.38855	1.35299	1.31684	1.28019	1.24313	1.20578
0.0400	1.48852	1.44813	1.40720	1.36584	1.32414	1.28225	1.24028
0.0250	1.61099	1.55914	1.50712	1.45507	1.40314	1.35153	1.30042
0.0200	1.66325	1.60604	1.54886	1.49188	1.43529	1.37929	1.32412
0.0100	1.80621	1.73271	1.66001	1.58838	1.51808	1.44942	1.38267
0.0050	1.92580	1.83660	1.74919	1.66390	1.58110	1.50114	1.42439
0.0020	2.05701	1.94806	1.84244	1.74062	1.64305	1.55016	1.46232
0.0010	2.14053	2.01739	1.89894	1.78572	1.67825	1.57695	1.48216
0.0005	2.21328	2.07661	1.94611	1.82241	1.70603	1.59738	1.49673
0.0001	2.35015	2.18448	2.02891	1.88410	1.75053	1.62838	1.51752

P	G =-1.4	G =-1.5	G =-1.6	G =-1.7	G =-1.8	G =-1.9	G =-2.0
0.9999	-6.86661	-7.09277	-7.31818	-7.54272	-7.76632	-7.98888	-8.21034
0.9995	-5.63252	-5.79673	-5.95990	-6.12196	-6.28285	-6.44251	-6.60090
0.9990	-5.09505	-5.23353	-5.37087	-5.50701	-5.64190	-5.77549	-5.90776
0.9980	-4.55304	-4.66651	-4.77875	-4.88971	-4.99937	-5.10768	-5.21461
0.9950	-3.82798	-3.90973	-3.99016	-4.06926	-4.14700	-4.22336	-4.29832
0.9900	-3.27134	-3.33035	-3.38804	-3.44438	-3.49935	-3.55295	-3.60517
0.9800	-2.70556	-2.74325	-2.77964	-2.81472	-2.84848	-2.88091	-2.91202
0.9750	-2.52102	-2.55222	-2.58214	-2.61076	-2.63810	-2.66413	-2.68888
0.9600	-2.12768	-2.14591	-2.16293	-2.17873	-2.19332	-2.20670	-2.21888
0.9500	-1.93836	-1.95083	-1.96213	-1.97227	-1.98124	-1.98906	-1.99573
0.9000	-1.33665	-1.33330	-1.32900	-1.32376	-1.31760	-1.31054	-1.30259
0.8000	-0.70512	-0.69050	-0.67532	-0.65959	-0.64335	-0.62662	-0.60944
0.7000	-0.31307	-0.29535	-0.27740	-0.25925	-0.24094	-0.22250	-0.20397
0.6000	-0.01824	-0.00092	0.01631	0.03344	0.05040	0.06718	0.08371
0.5704	0.05803	0.07476	0.09132	0.10769	0.12381	0.13964	0.15516
0.5000	0.22535	0.23996	0.25422	0.26808	0.28150	0.29443	0.30685
0.4296	0.37824	0.38977	0.40075	0.41116	0.42095	0.43008	0.43854
0.4000	0.43949	0.44942	0.45873	0.46739	0.47538	0.48265	0.48917
0.3000	0.63779	0.64080	0.64300	0.64436	0.64488	0.64453	0.64333
0.2000	0.83223	0.82516	0.81720	0.80837	0.79868	0.78816	0.77686
0.1000	1.04144	1.01810	0.99418	0.96977	0.94496	0.91988	0.89464
0.0500	1.16827	1.13075	1.09338	1.05631	1.01973	0.98381	0.94871
0.0400	1.19842	1.15682	1.11566	1.07513	1.03543	0.99672	0.95918
0.0250	1.25004	1.20059	1.15229	1.10537	1.06001	1.01640	0.97468
0.0200	1.26999	1.21716	1.16584	1.11628	1.06864	1.02311	0.97980
0.0100	1.31815	1.25611	1.19680	1.14042	1.08711	1.03695	0.98995
0.0050	1.35114	1.28167	1.21618	1.15477	1.09749	1.04427	0.99499
0.0020	1.37981	1.30279	1.23132	1.16534	1.10465	1.04898	0.99800
0.0010	1.39408	1.31275	1.23805	1.16974	1.10743	1.05068	0.99900
0.0005	1.40413	1.31944	1.24235	1.17240	1.10901	1.05159	0.99950
0.0001	1.41753	1.32774	1.24728	1.17520	1.11054	1.05239	0.99990

P	G =-2.1	G =-2.2	G =-2.3	G =-2.4	G =-2.5	G =-2.6	G1=-2.7
0.9999	-8.43064	-8.64971	-8.86753	-9.08403	-9.29920	-9.51301	-9.72543
0.9995	-6.75798	-6.91370	-7.06804	-7.22098	-7.37250	-7.52258	-7.67121
0.9990	-6.03865	-6.16816	-6.29626	-6.42292	-6.54814	-6.67191	-6.79421
0.9980	-5.32014	-5.42426	-5.52694	-5.62818	-5.72796	-5.82629	-5.92316
0.9950	-4.37186	-4.44398	-4.51467	-4.58393	-4.65176	-4.71815	-4.78313
0.9900	-3.65600	-3.70543	-3.75347	-3.80013	-3.84540	-3.88930	-3.93183
0.9800	-2.94181	-2.97028	-2.99744	-3.02330	-3.04787	-3.07116	-3.09320
0.9750	-2.71234	-2.73451	-2.75541	-2.77506	-2.79345	-2.81062	-2.82658
0.9600	-2.22986	-2.23967	-2.24831	-2.25581	-2.26217	-2.26743	-2.27160
0.9500	-2.00128	-2.00570	-2.00903	-2.01128	-2.01247	-2.01263	-2.01177
0.9000	-1.29377	-1.28412	-1.27365	-1.26240	-1.25039	-1.23766	-1.22422
0.8000	-0.59183	-0.57383	-0.55549	-0.53683	-0.51789	-0.49872	-0.47934
0.7000	-0.18540	-0.16682	-0.14827	-0.12979	-0.11143	-0.09323	-0.07523
0.6000	0.09997	0.11590	0.13148	0.14665	0.16138	0.17564	0.18939
0.5704	0.17030	0.18504	0.19933	0.21313	0.22642	0.23915	0.25129
0.5000	0.31872	0.32999	0.34063	0.35062	0.35992	0.36852	0.37640
0.4296	0.44628	0.45329	0.45953	0.46499	0.46966	0.47353	0.47660
0.4000	0.49494	0.49991	0.50409	0.50744	0.50999	0.51171	0.51263
0.3000	0.64125	0.63833	0.63456	0.62999	0.62463	0.61854	0.61176
0.2000	0.76482	0.75211	0.73880	0.72495	0.71067	0.69602	0.68111
0.1000	0.86938	0.84422	0.81929	0.79472	0.77062	0.74709	0.72422
0.0500	0.91458	0.88156	0.84976	0.81927	0.79015	0.76242	0.73610
0.0400	0.92295	0.88814	0.85486	0.82315	0.79306	0.76456	0.73765
0.0250	0.93495	0.89728	0.86169	0.82817	0.79667	0.76712	0.73943
0.0200	0.93878	0.90009	0.86371	0.82959	0.79765	0.76779	0.73987
0.0100	0.94607	0.90521	0.86723	0.83196	0.79921	0.76878	0.74049
0.0050	0.94945	0.90742	0.86863	0.83283	0.79973	0.76909	0.74067
0.0020	0.95131	0.90854	0.86929	0.83320	0.79994	0.76920	0.74073
0.0010	0.95188	0.90885	0.86945	0.83328	0.79998	0.76922	0.74074
0.0005	0.95215	0.90899	0.86952	0.83331	0.79999	0.76923	0.74074
0.0001	0.95234	0.90908	0.86956	0.83333	0.80000	0.76923	0.74074

P	G =-2.8	G =-2.9	G =-3.0	G =-3.1	G =-3.2	G =-3.3	G1=-3.4
0.9999	-9.93643	-10.14602	-10.35418	-10.56090	-10.76618	-10.97001	-11.17239
0.9995	-7.81839	-7.96411	-8.10836	-8.25115	-8.39248	-8.53236	-8.67079
0.9990	-6.91505	-7.03443	-7.15235	-7.26881	-7.38382	-7.49739	-7.60953
0.9980	-6.01858	-6.11254	-6.20506	-6.29613	-6.38578	-6.47401	-6.56084
0.9950	-4.84669	-4.90884	-4.96959	-5.02897	-5.08697	-5.14362	-5.19892
0.9900	-3.97301	-4.01286	-4.05138	-4.08859	-4.12452	-4.15917	-4.19257
0.9800	-3.11399	-3.13356	-3.15193	-3.16911	-3.18512	-3.20000	-3.21375
0.9750	-2.84134	-2.85492	-2.86735	-2.87865	-2.88884	-2.89795	-2.90599
0.9600	-2.27470	-2.27676	-2.27780	-2.27785	-2.27693	-2.27506	-2.27229
0.9500	-2.00992	-2.00710	-2.00335	-1.99869	-1.99314	-1.98674	-1.97951
0.9000	-1.21013	-1.19539	-1.18006	-1.16416	-1.14772	-1.13078	-1.11337
0.8000	-0.45980	-0.44015	-0.42040	-0.40061	-0.38081	-0.36104	-0.34133
0.7000	-0.05746	-0.03997	-0.02279	-0.00596	0.01050	0.02654	0.04215
0.6000	0.20259	0.21523	0.22726	0.23868	0.24946	0.25958	0.26904
0.5704	0.26282	0.27372	0.28395	0.29351	0.30238	0.31055	0.31802
0.5000	0.38353	0.38991	0.39554	0.40041	0.40454	0.40792	0.41058
0.4296	0.47888	0.48037	0.48109	0.48107	0.48033	0.47890	0.47682
0.4000	0.51276	0.51212	0.51073	0.50863	0.50585	0.50244	0.49844
0.3000	0.60434	0.59634	0.58783	0.57887	0.56953	0.55989	0.55000
0.2000	0.66603	0.65086	0.63569	0.62060	0.60567	0.59096	0.57652
0.1000	0.70209	0.68075	0.66023	0.64056	0.62175	0.60379	0.58666
0.0500	0.71116	0.68759	0.66532	0.64429	0.62445	0.60572	0.58802
0.0400	0.71227	0.68836	0.66585	0.64465	0.62469	0.60587	0.58812
0.0250	0.71348	0.68917	0.66638	0.64500	0.62491	0.60601	0.58821
0.0200	0.71377	0.68935	0.66649	0.64507	0.62495	0.60603	0.58822
0.0100	0.71415	0.68959	0.66663	0.64514	0.62499	0.60606	0.58823
0.0050	0.71425	0.68964	0.66666	0.64516	0.62500	0.60606	0.58824
0.0020	0.71428	0.68965	0.66667	0.64516	0.62500	0.60606	0.58824
0.0010	0.71428	0.68965	0.66667	0.64516	0.62500	0.60606	0.58824
0.0005	0.71429	0.68966	0.66667	0.64516	0.62500	0.60606	0.58824
0.0001	0.71429	0.68966	0.66667	0.64516	0.62500	0.60606	0.58824

P	G =-3.5	G =-3.6	G =-3.7	G =-3.8	G =-3.9	G =-4.0	G =-4.1
0.9999	-11.37334	-11.57284	-11.77092	-11.96757	-12.16280	-12.35663	-12.54906
0.9995	-8.80779	-8.94335	-9.07750	-9.21023	-9.34158	-9.47154	-9.60013
0.9990	-7.72024	-7.82954	-7.93744	-8.04395	-8.14910	-8.25289	-8.35534
0.9980	-6.64627	-6.73032	-6.81301	-6.89435	-6.97435	-7.05304	-7.13043
0.9950	-5.25291	-5.30559	-5.35698	-5.40711	-5.45598	-5.50362	-5.55005
0.9900	-4.22473	-4.25569	-4.28545	-4.31403	-4.34147	-4.36777	-4.39296
0.9800	-3.22641	-3.23800	-3.24853	-3.25803	-3.26653	-3.27404	-3.28060
0.9750	-2.91299	-2.91898	-2.92397	-2.92799	-2.93107	-2.93324	-2.93450
0.9600	-2.26862	-2.26409	-2.25872	-2.25254	-2.24558	-2.23786	-2.22940
0.9500	-1.97147	-1.96266	-1.95311	-1.94283	-1.93186	-1.92023	-1.90796
0.9000	-1.09552	-1.07726	-1.05863	-1.03965	-1.02036	-1.00079	-0.98096
0.8000	-0.32171	-0.30223	-0.28290	-0.26376	-0.24484	-0.22617	-0.20777
0.7000	0.05730	0.07195	0.08610	0.09972	0.11279	0.12530	0.13725
0.6000	0.27782	0.28592	0.29335	0.30010	0.30617	0.31159	0.31635
0.5704	0.32479	0.33085	0.33623	0.34092	0.34494	0.34831	0.35105
0.5000	0.41253	0.41381	0.41442	0.41441	0.41381	0.41265	0.41097
0.4296	0.47413	0.47088	0.46711	0.46286	0.45819	0.45314	0.44777
0.4000	0.49391	0.48888	0.48342	0.47758	0.47141	0.46496	0.45828
0.3000	0.53993	0.52975	0.51952	0.50929	0.49911	0.48902	0.47906
0.2000	0.56242	0.54867	0.53533	0.52240	0.50990	0.49784	0.48622
0.1000	0.57035	0.55483	0.54006	0.52600	0.51261	0.49986	0.48772
0.0500	0.57130	0.55548	0.54050	0.52629	0.51281	0.49999	0.48780
0.0400	0.57136	0.55552	0.54052	0.52630	0.51281	0.50000	0.48780
0.0250	0.57141	0.55555	0.54054	0.52631	0.51282	0.50000	0.48780
0.0200	0.57142	0.55555	0.54054	0.52631	0.51282	0.50000	0.48780
0.0100	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000	0.48780
0.0050	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000	0.48780
0.0020	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000	0.48780
0.0010	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000	0.48780
0.0005	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000	0.48780
0.0001	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000	0.48780

P	G =-4.2	G =-4.3	G =-4.4	G =-4.5	G =-4.6	G =-4.7	G =-4.8
0.9999	-12.74010	-12.92977	-13.11808	-13.30504	-13.49066	-13.67495	-13.85794
0.9995	-9.72737	-9.85326	-9.97784	-10.10110	-10.22307	-10.34375	-10.46318
0.9990	-8.45646	-8.55627	-8.65479	-8.75202	-8.84800	-8.94273	-9.03623
0.9980	-7.20654	-7.28138	-7.35497	-7.42733	-7.49847	-7.56842	-7.63718
0.9950	-5.59528	-5.63934	-5.68224	-5.72400	-5.76464	-5.80418	-5.84265
0.9900	-4.41706	-4.44009	-4.46207	-4.48303	-4.50297	-4.52192	-4.53990
0.9800	-3.28622	-3.29092	-3.29473	-3.29767	-3.29976	-3.30103	-3.30149
0.9750	-2.93489	-2.93443	-2.93314	-2.93105	-2.92818	-2.92455	-2.92017
0.9600	-2.22024	-2.21039	-2.19988	-2.18874	-2.17699	-2.16465	-2.15174
0.9500	-1.89508	-1.88160	-1.86757	-1.85300	-1.83792	-1.82234	-1.80631
0.9000	-0.96090	-0.94064	-0.92022	-0.89964	-0.87895	-0.85817	-0.83731
0.8000	-0.18967	-0.17189	-0.15445	-0.13737	-0.12067	-0.10436	-0.08847
0.7000	0.14861	0.15939	0.16958	0.17918	0.18819	0.19661	0.20446
0.6000	0.32049	0.32400	0.32693	0.32928	0.33108	0.33236	0.33315
0.5704	0.35318	0.35473	0.35572	0.35619	0.35616	0.35567	0.35475
0.5000	0.40881	0.40621	0.40321	0.39985	0.39617	0.39221	0.38800
0.4296	0.44212	0.43623	0.43016	0.42394	0.41761	0.41121	0.40477
0.4000	0.45142	0.44442	0.43734	0.43020	0.42304	0.41590	0.40880
0.3000	0.46927	0.45967	0.45029	0.44114	0.43223	0.42357	0.41517
0.2000	0.47504	0.46428	0.45395	0.44402	0.43448	0.42532	0.41652
0.1000	0.47614	0.46508	0.45452	0.44443	0.43477	0.42553	0.41666
0.0500	0.47619	0.46511	0.45454	0.44444	0.43478	0.42553	0.41667
0.0400	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667
0.0250	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667
0.0200	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667
0.0100	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667
0.0050	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667
0.0020	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667
0.0010	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667
0.0005	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667
0.0001	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667

P	G =-4.9	G =-5.0	G =-5.1	G =-5.2	G =-5.3	G =-5.4	G =-5.5
0.9999	-14.03963	-14.22004	-14.39918	-14.57706	-14.75370	-14.92912	-15.10332
0.9995	-10.58135	-10.69829	-10.81401	-10.92853	-11.04186	-11.15402	-11.26502
0.9990	-9.12852	-9.21961	-9.30952	-9.39827	-9.48586	-9.57232	-9.65766
0.9980	-7.70479	-7.77124	-7.83657	-7.90078	-7.96390	-8.02594	-8.08691
0.9950	-5.88004	-5.91639	-5.95171	-5.98602	-6.01934	-6.05169	-6.08307
0.9900	-4.55694	-4.57304	-4.58823	-4.60252	-4.61594	-4.62850	-4.64022
0.9800	-3.30116	-3.30007	-3.29823	-3.29567	-3.29240	-3.28844	-3.28381
0.9750	-2.91508	-2.90930	-2.90283	-2.89572	-2.88796	-2.87959	-2.87062
0.9600	-2.13829	-2.12432	-2.10985	-2.09490	-2.07950	-2.06365	-2.04739
0.9500	-1.78982	-1.77292	-1.75563	-1.73795	-1.71992	-1.70155	-1.68287
0.9000	-0.81641	-0.79548	-0.77455	-0.75364	-0.73277	-0.71195	-0.69122
0.8000	-0.07300	-0.05798	-0.04340	-0.02927	-0.01561	-0.00243	0.01028
0.7000	0.21172	0.21843	0.22458	0.23019	0.23527	0.23984	0.24391
0.6000	0.33347	0.33336	0.33284	0.33194	0.33070	0.32914	0.32729
0.5704	0.35343	0.35174	0.34972	0.34740	0.34481	0.34198	0.33895
0.5000	0.38359	0.37901	0.37428	0.36945	0.36453	0.35956	0.35456
0.4296	0.39833	0.39190	0.38552	0.37919	0.37295	0.36680	0.36076
0.4000	0.40177	0.39482	0.38799	0.38127	0.37469	0.36825	0.36196
0.3000	0.40703	0.39914	0.39152	0.38414	0.37701	0.37011	0.36345
0.2000	0.40806	0.39993	0.39211	0.38458	0.37734	0.37036	0.36363
0.1000	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0500	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0400	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0250	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0200	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0100	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0050	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0020	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0010	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0005	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0001	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364

P	G =-5.6	G =-5.7	G =-5.8	G =-5.9	G =-6.0	G =-6.1	G =-6.2
0.9999	-15.27632	-15.44813	-15.61878	-15.78826	-15.95660	-16.12380	-16.28989
0.9995	-11.37487	-11.48360	-11.59122	-11.69773	-11.80316	-11.90752	-12.01082
0.9990	-9.74190	-9.82505	-9.90713	-9.98815	-10.06812	-10.14706	-10.22499
0.9980	-8.14683	-8.20572	-8.26359	-8.32046	-8.37634	-8.43125	-8.48519
0.9950	-6.11351	-6.14302	-6.17162	-6.19933	-6.22616	-6.25212	-6.27723
0.9900	-4.65111	-4.66120	-4.67050	-4.67903	-4.68680	-4.69382	-4.70013
0.9800	-3.27854	-3.27263	-3.26610	-3.25898	-3.25128	-3.24301	-3.23419
0.9750	-2.86107	-2.85096	-2.84030	-2.82912	-2.81743	-2.80525	-2.79259
0.9600	-2.03073	-2.01369	-1.99629	-1.97855	-1.96048	-1.94210	-1.92343
0.9500	-1.66390	-1.64464	-1.62513	-1.60538	-1.58541	-1.56524	-1.54487
0.9000	-0.67058	-0.65006	-0.62966	-0.60941	-0.58933	-0.56942	-0.54970
0.8000	0.02252	0.03427	0.04553	0.05632	0.06662	0.07645	0.08580
0.7000	0.24751	0.25064	0.25334	0.25562	0.25750	0.25901	0.26015
0.6000	0.32519	0.32285	0.32031	0.31759	0.31472	0.31171	0.30859
0.5704	0.33573	0.33236	0.32886	0.32525	0.32155	0.31780	0.31399
0.5000	0.34955	0.34455	0.33957	0.33463	0.32974	0.32492	0.32016
0.4296	0.35484	0.34903	0.34336	0.33782	0.33242	0.32715	0.32202
0.4000	0.35583	0.34985	0.34402	0.33836	0.33285	0.32750	0.32230
0.3000	0.35700	0.35078	0.34476	0.33893	0.33330	0.32784	0.32256
0.2000	0.35714	0.35087	0.34483	0.33898	0.33333	0.32787	0.32258
0.1000	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258
0.0500	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258
0.0400	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258
0.0250	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258
0.0200	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258
0.0100	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258
0.0050	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258
0.0020	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258
0.0010	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258
0.0005	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258
0.0001	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258

P	G =-6.3	G =-6.4	G =-6.5	G =-6.6	G =-6.7	G =-6.8	G =-6.9
0.9999	-16.45487	-16.61875	-16.78156	-16.94329	-17.10397	-17.26361	-17.42221
0.9995	-12.11307	-12.21429	-12.31450	-12.41370	-12.51190	-12.60913	-12.70539
0.9990	-10.30192	-10.37785	-10.45281	-10.52681	-10.59986	-10.67197	-10.74316
0.9980	-8.53820	-8.59027	-8.64142	-8.69167	-8.74102	-8.78950	-8.83711
0.9950	-6.30151	-6.32497	-6.34762	-6.36948	-6.39055	-6.41086	-6.43042
0.9900	-4.70571	-4.71061	-4.71482	-4.71836	-4.72125	-4.72350	-4.72512
0.9800	-3.22484	-3.21497	-3.20460	-3.19374	-3.18241	-3.17062	-3.15838
0.9750	-2.77947	-2.76591	-2.75191	-2.73751	-2.72270	-2.70751	-2.69195
0.9600	-1.90449	-1.88528	-1.86584	-1.84616	-1.82627	-1.80618	-1.78591
0.9500	-1.52434	-1.50365	-1.48281	-1.46186	-1.44079	-1.41963	-1.39839
0.9000	-0.53019	-0.51089	-0.49182	-0.47299	-0.45440	-0.43608	-0.41803
0.8000	0.09469	0.10311	0.11107	0.11859	0.12566	0.13231	0.13853
0.7000	0.26097	0.26146	0.26167	0.26160	0.26128	0.26072	0.25995
0.6000	0.30538	0.30209	0.29875	0.29537	0.29196	0.28854	0.28511
0.5704	0.31016	0.30631	0.30246	0.29862	0.29480	0.29101	0.28726
0.5000	0.31549	0.31090	0.30639	0.30198	0.29766	0.29344	0.28931
0.4296	0.31702	0.31216	0.30743	0.30283	0.29835	0.29400	0.28977
0.4000	0.31724	0.31234	0.30757	0.30294	0.29844	0.29407	0.28982
0.3000	0.31745	0.31249	0.30769	0.30303	0.29850	0.29412	0.28985
0.2000	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986
0.1000	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986
0.0500	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986
0.0400	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986
0.0250	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986
0.0200	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986
0.0100	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986
0.0050	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986
0.0020	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986
0.0010	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986
0.0005	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986
0.0001	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986

P	G =-7.0	G =-7.1	G =-7.2	G =-7.3	G =-7.4	G =-7.5	G =-7.6
0.9999	-17.57979	-17.73636	-17.89193	-18.04652	-18.20013	-18.35278	-18.50447
0.9995	-12.80069	-12.89505	-12.98848	-13.08098	-13.17258	-13.26328	-13.35309
0.9990	-10.81343	-10.88281	-10.95129	-11.01890	-11.08565	-11.15154	-11.21658
0.9980	-8.88387	-8.92979	-8.97488	-9.01915	-9.06261	-9.10528	-9.14717
0.9950	-6.44924	-6.46733	-6.48470	-6.50137	-6.51735	-6.53264	-6.54727
0.9900	-4.72613	-4.72653	-4.72635	-4.72559	-4.72427	-4.72240	-4.71998
0.9800	-3.14572	-3.13263	-3.11914	-3.10525	-3.09099	-3.07636	-3.06137
0.9750	-2.67603	-2.65977	-2.64317	-2.62626	-2.60905	-2.59154	-2.57375
0.9600	-1.76547	-1.74487	-1.72412	-1.70325	-1.68225	-1.66115	-1.63995
0.9500	-1.37708	-1.35571	-1.33430	-1.31287	-1.29141	-1.26995	-1.24850
0.9000	-0.40026	-0.38277	-0.36557	-0.34868	-0.33209	-0.31582	-0.29986
0.8000	0.14434	0.14975	0.15478	0.15942	0.16371	0.16764	0.17123
0.7000	0.25899	0.25785	0.25654	0.25510	0.25352	0.25183	0.25005
0.6000	0.28169	0.27829	0.27491	0.27156	0.26825	0.26497	0.26175
0.5704	0.28355	0.27990	0.27629	0.27274	0.26926	0.26584	0.26248
0.5000	0.28528	0.28135	0.27751	0.27376	0.27010	0.26654	0.26306
0.4296	0.28565	0.28164	0.27774	0.27394	0.27025	0.26665	0.26315
0.4000	0.28569	0.28167	0.27776	0.27396	0.27026	0.26666	0.26315
0.3000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.2000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.1000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0500	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0400	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0250	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0200	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0100	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0050	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0020	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0010	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0005	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0001	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316

P	G =-7.7	G =-7.8	G =-7.9	G =-8.0	G =-8.1	G =-8.2	G =-8.3
0.9999	-18.65522	-18.80504	-18.95393	-19.10191	-19.24898	-19.39517	-19.54046
0.9995	-13.44202	-13.53009	-13.61730	-13.70366	-13.78919	-13.87389	-13.95778
0.9990	-11.28080	-11.34419	-11.40677	-11.46855	-11.52953	-11.58974	-11.64917
0.9980	-9.18828	-9.22863	-9.26823	-9.30709	-9.34521	-9.38262	-9.41931
0.9950	-6.56124	-6.57456	-6.58725	-6.59931	-6.61075	-6.62159	-6.63183
0.9900	-4.71704	-4.71358	-4.70961	-4.70514	-4.70019	-4.69476	-4.68887
0.9800	-3.04604	-3.03038	-3.01439	-2.99810	-2.98150	-2.96462	-2.94746
0.9750	-2.55569	-2.53737	-2.51881	-2.50001	-2.48099	-2.46175	-2.44231
0.9600	-1.61867	-1.59732	-1.57591	-1.55444	-1.53294	-1.51141	-1.48985
0.9500	-1.22706	-1.20565	-1.18427	-1.16295	-1.14168	-1.12048	-1.09936
0.9000	-0.28422	-0.26892	-0.25394	-0.23929	-0.22498	-0.21101	-0.19737
0.8000	0.17450	0.17746	0.18012	0.18249	0.18459	0.18643	0.18803
0.7000	0.24817	0.24622	0.24421	0.24214	0.24003	0.23788	0.23571
0.6000	0.25857	0.25544	0.25236	0.24933	0.24637	0.24345	0.24060
0.5704	0.25919	0.25596	0.25280	0.24970	0.24667	0.24371	0.24081
0.5000	0.25966	0.25635	0.25312	0.24996	0.24689	0.24388	0.24095
0.4296	0.25973	0.25640	0.25316	0.25000	0.24691	0.24390	0.24096
0.4000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.3000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.2000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.1000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0500	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0400	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0250	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0200	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0100	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0050	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0020	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0010	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0005	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0001	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096

P	G =-8.4	G =-8.5	G =-8.6	G =-8.7	G =-8.8	G =-8.9	G =-9.0
0.9999	-19.68489	-19.82845	-19.97115	-20.11300	-20.25402	-20.39420	-20.53356
0.9995	-14.04086	-14.12314	-14.20463	-14.28534	-14.36528	-14.44446	-14.52288
0.9990	-11.70785	-11.76576	-11.82294	-11.87938	-11.93509	-11.99009	-12.04437
0.9980	-9.45530	-9.49060	-9.52521	-9.55915	-9.59243	-9.62504	-9.65701
0.9950	-6.64148	-6.65056	-6.65907	-6.66703	-6.67443	-6.68130	-6.68763
0.9900	-4.68252	-4.67573	-4.66850	-4.66085	-4.65277	-4.64429	-4.63541
0.9800	-2.93002	-2.91234	-2.89440	-2.87622	-2.85782	-2.83919	-2.82035
0.9750	-2.42268	-2.40287	-2.38288	-2.36273	-2.34242	-2.32197	-2.30138
0.9600	-1.46829	-1.44673	-1.42518	-1.40364	-1.38213	-1.36065	-1.33922
0.9500	-1.07832	-1.05738	-1.03654	-1.01581	-0.99519	-0.97471	-0.95435
0.9000	-0.18408	-0.17113	-0.15851	-0.14624	-0.13431	-0.12272	-0.11146
0.8000	0.18939	0.19054	0.19147	0.19221	0.19277	0.19316	0.19338
0.7000	0.23352	0.23132	0.22911	0.22690	0.22469	0.22249	0.22030
0.6000	0.23779	0.23505	0.23236	0.22972	0.22714	0.22461	0.22214
0.5704	0.23797	0.23520	0.23248	0.22982	0.22722	0.22468	0.22219
0.5000	0.23808	0.23528	0.23255	0.22988	0.22727	0.22472	0.22222
0.4296	0.23809	0.23529	0.23256	0.22988	0.22727	0.22472	0.22222
0.4000	0.23810	0.23529	0.23256	0.22988	0.22727	0.22472	0.22222
0.3000	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.2000	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.1000	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0500	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0400	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0250	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0200	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0100	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0050	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0020	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0010	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0005	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0001	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222

*

OUTLIER TEST K VALUES

10 PERCENT SIGNIFICANCE LEVEL K VALUES

The table below contains one sided 10 percent significance level K_N values for a normal distribution (38). Tests conducted to select the outlier detection procedures used in this report indicate these K_N values are applicable to log-Pearson Type III distributions over the tested range of skew values.

Sample size	K_N value	Sample size	K_N value	Sample size	K_N value	Sample size	K_N value
10	2.036	45	2.727	80	2.940	115	3.064
11	2.088	46	2.736	81	2.945	116	3.067
12	2.134	47	2.744	82	2.949	117	3.070
13	2.175	48	2.753	83	2.953	118	3.073
14	2.213	49	2.760	84	2.957	119	3.075
15	2.247	50	2.768	85	2.961	120	3.078
16	2.279	51	2.775	86	2.966	121	3.081
17	2.309	52	2.783	87	2.970	122	3.083
18	2.335	53	2.790	88	2.973	123	3.086
19	2.361	54	2.798	89	2.977	124	3.089
20	2.385	55	2.804	90	2.981	125	3.092
21	2.408	56	2.811	91	2.984	126	3.095
22	2.429	57	2.818	92	2.989	127	3.097
23	2.448	58	2.824	93	2.993	128	3.100
24	2.467	59	2.831	94	2.996	129	3.102
25	2.486	60	2.837	95	3.000	130	3.104
26	2.502	61	2.842	96	3.003	131	3.107
27	2.519	62	2.849	97	3.006	132	3.109
28	2.534	63	2.854	98	3.011	133	3.112
29	2.549	64	2.860	99	3.014	134	3.114
30	2.563	65	2.866	100	3.017	135	3.116
31	2.577	66	2.871	101	3.021	136	3.119
32	2.591	67	2.877	102	3.024	137	3.122
33	2.604	68	2.883	103	3.027	138	3.124
34	2.616	69	2.888	104	3.030	139	3.126
35	2.628	70	2.893	105	3.033	140	3.129
36	2.639	71	2.897	106	3.037	141	3.131
37	2.650	72	2.903	107	3.040	142	3.133
38	2.661	73	2.908	108	3.043	143	3.135
39	2.671	74	2.912	109	3.046	144	3.138
40	2.682	75	2.917	110	3.049	145	3.140
41	2.692	76	2.922	111	3.052	146	3.142
42	2.700	77	2.927	112	3.055	147	3.144
43	2.710	78	2.931	113	3.058	148	3.146
44	2.719	79	2.935	114	3.061	149	3.148

*

* Appendix 5

CONDITIONAL PROBABILITY ADJUSTMENT

For stations where the record of annual peaks is truncated by the omission of peaks below a gage base, years with zero flow, and/or low outlier criterion, the conditional probability adjustment described in reference (28) is recommended to obtain the frequency curve. These procedures should only be used when not over 25 percent of the total record has been truncated. A truncation level is defined as the minimum discharge that will exclude peaks below the gage base, zero flows, all low outliers, and no other discharges. Because data from stations treated by this procedure may not fit a log-Pearson Type III distribution, any computed frequency curve should be compared with a plot of observed values.

Prior to applying the conditional probability adjustment, the data should have been reviewed and the statistics for the above gage-base peaks computed. Procedures for detecting outliers, recomputing statistics for peaks above the truncation level, and incorporating applicable historic information should have been completed. All except the last computation step shown on the flow chart in Appendix 12 (page 12-3) should have been completed. The steps in the conditional probability adjustment are as follows:

1. Calculate the estimated probability $\tilde{P}$ that any annual peak will exceed the truncation level by the formula:

$$\tilde{P} = \frac{N}{n} \qquad (5\text{-}1a)$$

in which N is the number of peaks above the truncation level and n is the total number of years of record. If historic information has been included, then equation 5-1b should be used rather than 5-1a. *

*

$$\tilde{P} = \frac{H-WL}{H} \qquad (5\text{-}1b)$$

where H is the historic record length, L the number of peaks truncated and W the systematic record weight as computed in Appendix 6, equation 6-1.

2. Recompute the exceedance probabilities, P, for selected points, P_d, on the frequency curve using equation 5-2:

$$P = \tilde{P} \times P_d \qquad (5\text{-}2)$$

This accounts for the omission of peaks below the truncation level.

3. The exceedance probabilities, P, computed by equation 5-2 are usually not those needed to compute the synthetic sample statistics. Therefore, it is necessary to interpolate either graphically or mathematically to obtain log discharge values for the 0.01, 0.10, and 0.50 exceedance probabilities.

4. Since the conditional probability adjusted frequency curve does not have known statistics, synthetic ones will be computed. These synthetic statistics will be determined based on the values for the three exceedance probabilities determined in step 3, using the following equations.

$$G_s = -2.50 + 3.12 \frac{\text{Log}(Q_{.01}/Q_{.10})}{\text{Log}(Q_{.10}/Q_{.50})} \qquad (5\text{-}3)$$

$$S_s = \frac{\text{Log}\ (Q_{.01}/Q_{.50})}{K_{.01} - K_{.50}} \qquad (5\text{-}4)$$

$$\overline{X}_s = \text{Log}(Q_{.50}) - K_{.50}(S_s) \qquad (5\text{-}5)$$

where G_s, S_s, and $\overline{X}_s$ are the synthetic logarithmic skew coefficient, standard deviation, and mean, respectively; $Q_{.01}$, $Q_{.10}$, and $Q_{.50}$ are discharges

*

*with 0.01, and 0.10, and 0.50 exceedance probabilities respectively; and $K_{.01}$ and $K_{.50}$ are Pearson Type III deviates for exceedance probabilities of 0.01 and 0.50 respectively, and skew coefficient G_s. Equation 5-3 is an approximation appropriate for use between skew values of +2.5 and -2.0.

5. The frequency curve developed from the synthetic statistics should be compared with the observed annual peak discharges. The plotting position should be based upon the total number of years record, n or H, as appropriate.

The minimum additional requirement to arrive at a final frequency curve is the determination of the weighted skew. Examples 3 and 4 of Appendix 12 illustrate the basic steps in computing a frequency curve using the conditional probability adjustment. Other considerations in a complete analysis might include two-station comparison, use of rainfall data, or other techniques described in this report. *

*

NOTATION

G_s	= synthetic logarithmic skew coefficient
H	= historic record length
$K_{.01}$, $K_{.50}$	= Pearson type III deviate from Appendix 3 for exceedance probabilities of 0.01 and 0.50 respectively, and skew coefficient G_s.
L	= number of peaks truncated
N	= number of peaks above the truncation level
n	= total number of years of record
P	= exceedance probabilities
$\tilde{P}$	= estimated probability that an annual peak will exceed the truncation level.
P_d	= selected points on the frequency curve
$Q_{.01}$, $Q_{.10}$, $Q_{.50}$	= discharges with exceedance probabilities of 0.01, 0.10, and 0.50, respectively
S_s	= synthetic logarithmic standard deviation
W	= systematic record weight from Appendix 6
$\overline{X}_s$	= synthetic logarithmic mean

*

Appendix 6
HISTORIC DATA

Flood information outside that in the systematic record can often be used to extend the record of the largest events to a historic period much longer than that of the systematic record. In such a situation, the following analytical techniques are used to compute a historically adjusted log-Pearson Type III frequency curve.

1. Historic knowledge is used to define the historically longer period of "H" years. The number "Z" of events that are known to be the largest in the historically longer period "H" are given a weight of 1.0. The remaining "N" events from the systematic record are given a weight of (H-Z)/(N+L) on the assumption that their distribution is representative of the (H-Z) remaining years of the historically longer period.

2. The computations can be done directly by applying the weights to each individual year's data using equations 6-1, 6-2a, 6-3a, and 6-4a. Figure 6-1 is an example of this procedure in which there are 44 years of systematic record and the 1897, 1919 and 1927 floods are known to be the three largest floods in the 77 year period 1897 to 1973. If statistics have been previously computed for the current continuous record, they can be adjusted to give the equivalent historically adjusted values using equations 6-1, 6-2b, 6-3b, and 6-4b, as illustrated in Figure 6-2.

3. The historically adjusted frequency curve is sketched on logarithmic-probability paper through points established by use of equation 6-5. The individual flood events should also be plotted for comparison. The historically adjusted plotting positions for the individual flood events are computed by use of equation 6-8, in which the historically adjusted order number of each event "$\tilde{m}$" is computed from equations 6-6 and 6-7. The computations are illustrated in Figures 6-1 and 6-2, and the completed plotting is shown in Figure 6-3.

4. The following example illustrates the steps in application of the historic peak adjustment only. It does not include the final step of weighting with the generalized skew. The historically adjusted skew developed by this procedure is appropriate to use in developing a generalized skew.

DEFINITION OF SYMBOLS

E = event number when events are ranked in order from greatest magnitude to smallest magnitude. The event numbers "E" will range from 1 to (Z + N).

X = logarithmic magnitude of systematic peaks excluding zero flood events, peaks below base, high or low outliers

X_z = logarithmic magnitude of a historic peak including a high outlier that has historic information

N = number of X's

M = mean of X's

$\tilde{M}$ = historically adjusted mean

$\tilde{m}$ = historically adjusted order number of each event for use in formulas to compute the plotting position on probability paper

S = standard deviation of the X's

$\tilde{S}$ = historically adjusted standard deviation

G = skew coefficient of the X's

$\tilde{G}$ = historically adjusted skew coefficient

K = Pearson Type III coordinate expressed in number of standard deviations from the mean for a specified recurrence interval or percent chance

Q = computed flood flow for a selected recurrence interval or percent chance

$\widetilde{PP}$ = plotting position in percent

$\tilde{P}$ = probability that any peak will exceed the truncation level (used in step 1, Appendix 5)

Z = number of historic peaks including high outliers that have historic information

H = number of years in historic period

L = number of low values to be excluded, such as: number of zeros, number of incomplete record years (below measurable base), and low outliers which have been identified

a = constant that is characteristic of a given plotting position formula. For Weibull formula, a = 0; for Beard formula, a = 0.3; and for Hazen formula, a = 0.5

W = systematic record weight

EQUATIONS

$$W = \frac{H - Z}{N + L} \tag{6-1}$$

$$\tilde{M} = \frac{W\Sigma X + \Sigma X_Z}{H\text{-}WL} \tag{6-2a}$$

$$\tilde{S}^2 = \frac{W\Sigma(X - \tilde{M})^2 + \Sigma(X_Z - \tilde{M})^2}{(H\text{-}WL\text{-}1)} \tag{6-3a}$$

$$\tilde{G} = \frac{H\text{-}WL}{(H\text{-}WL\text{-}1)\ (H\text{-}WL\text{-}2)}\left[\frac{W\Sigma(X - \tilde{M})^3 + \Sigma(X_Z - \tilde{M})^3}{\tilde{S}^3}\right] \tag{6-4a}$$

$$\tilde{M} = \frac{WNM + \Sigma X_Z}{H\text{-}WL} \tag{6-2b}$$

$$\tilde{S}^2 = \frac{W\ (N - 1)S^2 + WN\ (M - \tilde{M})^2 + \Sigma(X_Z - \tilde{M})^2}{(H\text{-}WL\text{-}1)} \tag{6-3b}$$

$$\tilde{G} = \frac{H\text{-}WL}{(H\text{-}WL\text{-}1)\ (H\text{-}WL\text{-}2)\tilde{S}^3}\left[\frac{W(N - 1)\ (N - 2)S^3G}{N} + 3W\ (N - 1)\ (M - \tilde{M})S^2 + WN\ (M - \tilde{M})^3 + \Sigma(X_Z - \tilde{M})^3\right] \tag{6-4b}$$

$$\text{Log}\ \ Q = \tilde{M} + K\tilde{S} \tag{6-5}$$

$$\tilde{m} = E;\ \text{when: } 1 \leq E \leq Z \tag{6-6}$$

$$\tilde{m} = WE - (W - 1)\ (Z + 0.5);\ \text{when: } (Z + 1) \leq E \leq (Z + N{+}L) \tag{6-7}$$

$$P\tilde{P} = \frac{\tilde{m} - a}{H + 1 - 2a}\ 100 \tag{6-8}$$

Figure 6-1. HISTORICALLY WEIGHTED LOG PEARSON TYPE III - ANNUAL PEAKS

Station: 3-6065, Big Sandy River at Bruceton, TN. D. A. 205 square miles
Record: 1897, 1919, 1927, 1930-1973 (47 years)
Historical period: 1897-1973 (77 years)
N = 44; Z = 3; H = 77

Year	Q (ft^3/s) = Y	Log Y = X	Departure from mean log x = ($X-\bar{M}$)	Weight = W	Event Number = E	Weighted order Number = m	Plotting position (Weibull) PP
1897	25,000	4.39794	0.68212	1.00	1	1.00	1.28
1919	21,000	4.32222	0.60640	1.00	2	2.00	2.56
1927	18,500	4.26717	0.55136	1.00	3	3.00	3.85
1935	17,000	4.23045	0.51464	1.68182	4	4.34	5.56
1937	13,800	4.13988	0.42407		5	6.02	7.72
1946	12,000	4.07918	0.36337		6	7.71	9.88
1972	12,000	4.07918	0.36337		7	9.39	12.04
1956	11,800	4.07188	0.35607		8	11.07	14.19
1942	10,100	4.00432	0.28851		9	12.75	16.35
1950	9,880	3.99475	0.27895		10	14.43	18.50
1930	9,100	3.95904	0.24323		11	16.12	20.67
1967	9,060	3.95713	0.24132		12	17.80	22.82
1932	7,820	3.89321	0.17740		13	19.48	24.97
1973	7,640	3.88309	0.16728		14	21.16	27.13
1962	7,480	3.87390	0.15809		15	22.84	29.28
1965	7,180	3.85612	0.14031		16	24.53	31.45
1936	6,740	3.82866	0.11285		17	26.21	33.60
1948	6,130	3.78746	0.07165		18	27.89	35.76
1939	5,940	3.77379	0.05798		19	29.57	37.91
1945	5,630	3.75051	0.03470		20	31.25	40.06
1934	5,580	3.74663	0.03082		21	32.94	42.23
1955	5,480	3.73878	0.02297		22	34.62	44.38
1944	5,340	3.72754	0.01173		23	36.30	46.54
1951	5,230	3.71850	0.00269		24	37.98	48.69
1957	5,150	3.71181	-0.00400		25	39.66	50.85
1971	5,080	3.70586	-0.00995		26	41.35	53.01
1953	5,000	3.69897	-0.01684		27	43.03	55.17
1949	4,740	3.67578	-0.04003		28	44.71	57.32
1970	4,330	3.63649	-0.07932		29	46.39	59.47
1938	4,270	3.63043	-0.08538		30	48.07	61.62
1952	4,260	3.62941	-0.08640		31	49.76	63.79
1947	3,980	3.59988	-0.11593		32	51.44	65.95
1943	3,780	3.57749	-0.13832		33	53.12	68.10
1961	3,770	3.57634	-0.13947		34	54.80	70.25
1958	3,350	3.52504	-0.19077		35	56.49	72.42
1954	3,320	3.52114	-0.19467		36	58.17	74.58
1933	3,220	3.50786	-0.20795		37	59.85	76.73
1964	3,100	3.49136	-0.22445		38	61.53	78.88
1968	3,080	3.48855	-0.22725		39	63.21	81.04
1969	2,800	3.44716	-0.26865		40	64.90	83.21
1963	2,740	3.43775	-0.27806		41	66.58	85.36
1959	2,400	3.38021	-0.33560		42	68.26	87.51
1931	2,060	3.31387	-0.40194		43	69.94	89.67
1966	1,920	3.28330	-0.43251		44	71.62	91.82
1940	1,680	3.22531	-0.49050		45	73.31	93.99
1960	1,460	3.16435	-0.55146		46	74.99	96.14
1941	1,200	3.07918	-0.63663	1.68182	47	76.67	98.29

$W = (H - Z)N = (77 - 3)/(44) = 1.68182$

* Figure 6-1. HISTORICALLY WEIGHTED LOG PEARSON-TYPE III - ANNUAL PEAKS (Continued)

Solving (Eq. 6-2a)

$\Sigma X = 162.40155$

$W\Sigma X = 273.13018$

$\Sigma X_z = \underline{12.98733}$

286.11751

$\tilde{M} = 286.11751/77 = \underline{3.71581}$

Solving (Eq. 6-3a)

$\Sigma x^2 = 3.09755$

$W\Sigma x^2 = 5.20952$

$\Sigma x_z^2 = \underline{1.13705}$

6.34657

$\tilde{S}^2 = 6.34657/(77 - 1) = 0.08351$

$\tilde{S} = \underline{0.28898} \qquad \tilde{S}^3 = 0.02413$

Solving (Eq. 6-4a)

$\Sigma x^3 = -0.37648$

$W\Sigma x^3 = -0.63317$

$\Sigma x_z^3 = \underline{0.70802}$

0.07485

$$\tilde{G} = \frac{(77)\ (0.07485)}{(76)\ (75)\ (0.02413)} = \underline{0.0418}$$

Solving (Eq. 6, Page 13)

N = 77

A = -0.33 + 0.08 (0.0418) = -0.32666

B = 0.94 - 0.26 (0.0418) = 0.92913

$$MSE_G = 10^{[-0.32666\ -\ 0.92913[0.88649]]} = 10^{[-1.150325]} = 0.07074$$

Solving (Eq. 9.5, Page 12)

$$G_w = \frac{0.302(0.0418) + 0.07074(-0.2)}{.302 + 0.07074} = -0.00409$$

Solving (Eq. 6-5)

%	K $G_w = -0.00409$	$(S)\ (K)$ $\tilde{S} = .28898$	$\tilde{M} + (\tilde{S})\ (K) = \text{Log } Q$ $\tilde{M} = 3.71581$	Q (ft^3/s)
99	-2.32934	-0.67313	3.04269	1,103
95	-1.64599	-0.47566	3.24014	1,738
90	-1.28196	-0.37046	3.34535	2,215
80	-0.84141	-0.24315	3.47266	2,969
50	0.00067	0.00019	3.71600	5,200
20	0.84180	0.24326	3.95907	9,100
10	1.28110	0.37021	4.08602	12,190
4	1.74929	0.50551	4.22132	16,646
2	2.05159	0.59289	4.30868	20,355
1	2.32340	0.67142	4.38723	24,391
.1	3.08455	0.89138	4.60719	40,475
.01	3.71054	1.07227	4.78808	61,387

Solving (Eq. 6-6)

Z = 3

For E = 1; $\tilde{m}$ = E = 1

For E = 2; $\tilde{m}$ = E = 2

For E = 3; $\tilde{m}$ = E = 3

Solving (Eq. 6-8)

For Weibull: a = 0. $\widetilde{PP} = (100)\ (\tilde{m})/(78)$

Solving (Eq. 6-7)

(Z + 1) = 4

(Z + N) = 47

For $4 \leq E \leq 47$:

$\tilde{m} = (1.682)\ (E) - (0.682)\ (3.5)$

$\tilde{m} = (1.682)\ (E) - 2.387$

*

* Figure 6-2. HISTORICALLY WEIGHTED LOG-PEARSON TYPE III - ANNUAL PEAKS

Results of Standard Computation for the Current Continuous Record

Big Sandy River at Bruceton, TN. DA - 205 square miles
#3-6065 (44 years)

N = number of observations used = 44

M = mean of logarithms = 3.69094

S = standard deviation of logarithms = 0.26721

S^2 = 0.07140 S^3 = 0.01908

G = coefficient of skewness (logs) = -0.18746

Adjustment to Historically Weighted 77 Years

Historic Peaks (Z = 3 Years)					
Year	Y_z (ft^3/s)	Log $Y_z = X_z$	$X_z - \tilde{M}$	$(X_z - \tilde{M})^2$	$(X_z - \tilde{M})^3$
1897	25,000	4.39794	0.68213	0.46531	0.31740
1919	21,000	4.32222	0.60641	0.36774	0.22300
1927	18,500	4.26717	0.55136	0.30400	0.16762
Summation		12.98733	1.83990	1.13705	0.70802

N = 44 Z = 3 H = 77

Solving (Eq. 6-1): $W = (77-3)/44 = 1.68182$

Solving (Eq. 6-2b): $\tilde{M} = \dfrac{(1.68182)\ (44)\ (3.69094) + (12.98733)}{77} = 3.71581$

Solving (Eq. 6-3b):

$(M - \tilde{M}) = -0.02487$; $(M - \tilde{M})^2 = 0.000619$; $(M - \tilde{M})^3 = -0.0000154$

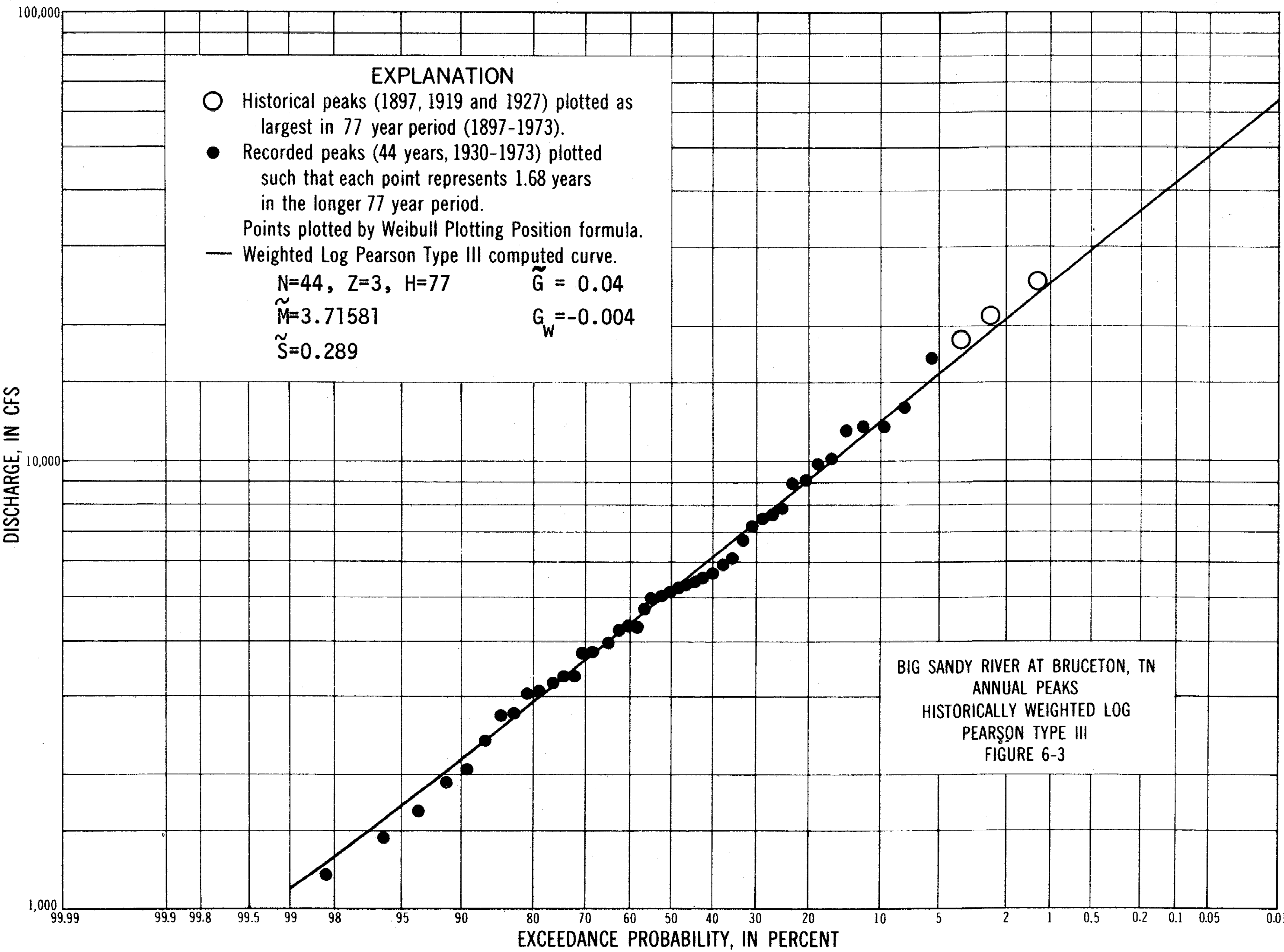
$$\tilde{S}^2 = \frac{(1.68182)(43)(0.07140) + (1.68182)(44)(0.000619) + (1.13705)}{76} = 0.08351$$

$\tilde{S}^2 = 0.08351$ $\tilde{S} = 0.28898$ $\tilde{S}^3 = 0.02413$

Solving (Eq. 6-4b):

$$\tilde{G} = \frac{77}{(76)\ (75)\ (0.02413)} \left[\frac{(1.68182)\ (43)\ \ (42)\ \ (0.01908)\ \ (-0.18746)}{44} + (3)(1.68182)(43)(-0.02487)(0.07140) + (1.68182)(44)(-0.0000154) + (0.70802)\right]$$

$\tilde{G} = 0.0418$ *


EXPLANATION
Historical peaks (1897, 1919 and 1927) plotted as largest in 77 year period (1897-1973).
Recorded peaks (44 years, 1930-1973) plotted such that each point represents 1.68 years in the longer 77 year period.
Points plotted by Weibull Plotting Position formula.
Weighted Log Pearson Type III computed curve.
N=44, Z=3, H=77
$\tilde{M}$=3.71581
$\tilde{S}$=0.289
$\tilde{G}$ = 0.04
G_W=-0.004
BIG SANDY RIVER AT BRUCETON, TN
ANNUAL PEAKS
HISTORICALLY WEIGHTED LOG
PEARSON TYPE III
FIGURE 6-3
DISCHARGE, IN CFS
100,000
10,000
1,000
99.99
99.9
99.8
99.5
99
98
95
90
80
70
60
50
40
30
20
10
5
2
1
0.5
0.2
0.1
0.05
0.01
EXCEEDANCE PROBABILITY, IN PERCENT

*

Appendix 7

TWO STATION COMPARISON

INTRODUCTION

The procedure outlined herein is recommended for use in adjusting the logarithmic mean and standard deviation of a short record on the basis of a regression analysis with a nearby long-term record. The theoretical basis for the equations provided herein were developed by Matalas and Jacobs (29).

The first step of the procedure is to correlate observed peak flows for the short record with concurrent observed peak flows for the long record. The regression and correlation coefficients, respectively, can be computed by the following two equations:

$$b = \frac{\Sigma X_1 Y_1 - \Sigma X_1 \Sigma Y_1 / N_1}{\Sigma X_1^2 - (\Sigma X_1)^2 / N_1} \qquad (7\text{-}1)$$

$$r = b \frac{S_{x_1}}{S_{y_1}} \qquad (7\text{-}2)$$

where the terms are defined at the end of this Appendix.

If the correlation coefficient defined by equation 7-2 meets certain criteria, then improved estimates of the short record mean and standard deviation can be made. Both of these statistics can be improved when the variance of that statistic is reduced. As each statistic is evaluated separately, only one adjustment may be worthwhile. The criterion and adjustment procedure for each statistic are discussed separately. In each discussion, two cases are considered: (1) entire short record contained in the long record, (2) only part of the short record contained in the long record. The steps for case 2 include all of those for case 1 plus an additional one.

*

The variance of the adjusted mean $(\overline{Y})$ can be determined by equation 7-3:

$$\operatorname{Var}(\overline{Y}) = \frac{\left(S_{y_1}\right)^2}{N_1} \left[1 - \frac{N_2}{N_1 + N_2} \left(r^2 - \frac{(1-r^2)}{(N_1-3)} \right) \right] \qquad (7\text{-}3)$$

Since $(S_{y_1})^2/N_1$ is the variance of $\overline{Y}_1$, the short-record mean, $\overline{Y}$ will be a better estimate of the true mean than $\overline{Y}_1$ if the term $r^2 - \frac{1-r^2}{N_1-3}$ in equation 7-3 is positive. Solving this relationship for r yields equation 7-4. If the correlation coefficient satisfies equation 7-4,

$$r > 1/(N_1 - 2)^{1/2} \qquad (7\text{-}4)$$

then an adjustment to the mean is worthwhile. The right side of this inequality represents the minimum critical value of r. Table 7-1 contains minimum critical values of r for various values of N_1. The adjusted logarithmic mean can be computed using equation 7-5a or 7-5b.

$$\overline{Y} = \overline{Y}_1 + \frac{N_2}{N_1+N_2} \left[b \quad (\overline{X}_2 - \overline{X}_1) \right] \qquad (7\text{-}5a)$$

$$\overline{Y} = \overline{Y}_1 + b(\overline{X}_3 - \overline{X}_1) \qquad (7\text{-}5b)$$

Equation 7-5b saves recomputing a new $\overline{X}_2$ at the long record station for each short record station that is being correlated with the long record station. While the adjusted mean from equation 7-5a or 7-5b may be an improved estimate of the mean obtained from the concurrent period, it may not be an improvement over the entire short record mean in case 2. It is necessary to compare the variance of the adjusted mean (equation 7-3) to the variance of the mean $(\overline{Y}_3)$ for the entire short record period (N_3). Compute the variance of the mean $\overline{Y}_3$ using equation 7-6:

$$\operatorname{Var}(\overline{Y}_3) = \frac{\left(S_{y_3}\right)^2}{N_3} \qquad (7\text{-}6)$$

*

*

where S_{y_3} is the standard deviation of the logarithms of flows for the short record site for the period N_3. If the variance of equation 7-6 is smaller than the variance of $\overline{Y}$ given in equation 7-3, then use $\overline{Y}_3$ as the final estimate of the mean. Otherwise, use the value of $\overline{Y}$ computed in equation 7-5a or 7-5b.

EQUIVALENT YEARS OF RECORD FOR THE MEAN

As illustrated in equations 7-3 and 7-6, the variance of the mean is inversely proportional to the record length at the site. Using equation 7-3 it can be shown that the equivalent years of record, N_e, for the adjusted mean is:

$$N_e = \frac{N_1}{1 - \frac{N_2}{N_1+N_2}\left(r^2 - \frac{(1-r^2)}{(N_1-3)} \right)} \qquad (7\text{-}7)$$

It may be seen from equation 7-7 that when there is no correlation (r=0), then N_e is less than N_1. This indicates that the correlation technique can actually decrease the equivalent years of record unless r satisfies equation 7-4. For perfect correlation (r=1), then $N_e = N_1 + N_2$, the total record length at the long record site.

Although N_e is actually the equivalent years of record for the mean, it is recommended that N_e be used as an estimate of the equivalent years of record for the various exceedance probability floods in the computation of confidence limits and in applying the expected probability adjustment.

CRITERION AND ADJUSTMENT PROCEDURE FOR THE STANDARD DEVIATION

The variance of the adjusted variance S_y^2 (square of the standard deviation) can be determined by equation 7-8:

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$$\mathrm{Var}(S_y^{\,2}) = \frac{2(S_{y_1})^4}{N_1-1} + \frac{N_2(S_{y_1})^4}{(N_1+N_2-1)^2}\left[Ar^4 + Br^2 + C\right] \qquad (7\text{-}8)$$

where A, B, and C are defined below and the other terms are defined at the end of the appendix. In equation 7-8, $2(S_{y_1})^4/(N_1-1)$ is the variance of $S_{y_1}^{\;2}$ (the short-record variance). If the second term in equation 7-8 is negative, then the variance of $S_y^{\,2}$ will be less than the variance of $S_{y_1}^{\;2}$. Solving this relationship for r yields the following equation:

$$|r| > \left[\frac{-B \pm \sqrt{B^2-4AC}}{2A}\right]^{1/2} \qquad (7\text{-}9)$$

where

$$A = \frac{(N_2+2)(N_1-6)(N_1-8)}{(N_1-3)(N_1-5)} - \frac{8(N_1-4)}{(N_1-3)} - \frac{2N_2(N_1-4)^2}{(N_1-3)^2} + \frac{N_1N_2(N_1-4)^2}{(N_1-3)^2(N_1-2)} + \frac{4(N_1-4)}{(N_1-3)}$$

$$B = \frac{6(N_2+2)(N_1-6)}{(N_1-3)(N_1-5)} + \frac{2(N_1^2-N_1-14)}{(N_1-3)} + \frac{2N_2(N_1-4)(N_1-5)}{(N_1-3)^2} - \frac{2(N_1-4)(N_1+3)}{(N_1-3)} - \frac{2N_1N_2(N_1-4)^2}{(N_1-3)^2(N_1-2)}$$

$$C = \frac{2(N_1+1)}{N_1-3} + \frac{3(N_2+2)}{(N_1-3)(N_1-5)} - \frac{(N_1+1)(2N_1+N_2-2)}{N_1-1} + \frac{2N_2(N_1-4)}{(N_1-3)^2} + \frac{2(N_1-4)(N_1+1)}{(N_1-3)} + \frac{N_1N_2(N_1-4)^2}{(N_1-3)^2(N_1-2)}$$

*

*The right side of the inequality (7-9) represents the minimum critical value of r. Table 7-1 gives approximate minimum critical values of r for various values of N_1. The table values are an approximation as they are solutions of equation 7-9 for a constant N_2. The variations in N_2 only affect the table values slightly.

If the correlation coefficient satisfies equation 7-9, then the adjusted variance can be computed by equation 7-10:

$$S_y{}^2 = \frac{1}{(N_1+N_2-1)} \Big[(N_1-1)S_{y_1}{}^2 + (N_2-1)b^2 S_{x_2}{}^2$$

$$+ \frac{N_2(N_1-4)(N_1-1)}{(N_1-3)(N_1-2)} (1-r^2)S_{y_1}{}^2 + \frac{N_1 N_2}{N_1+N_2} b^2 (\bar{X}_2 - \bar{X}_1)^2 \Big] \quad (7\text{-}10)$$

The adjusted standard deviation S_y equals the square root of the adjusted variance in equation 7-10. The third term in brackets in equation 7-10 is an adjustment factor to give an unbiased estimate of $S_y{}^2$. This adjustment is equivalent to adding random noise to each estimated value of flow at the short-term site.

While the adjusted variance from equation 7-10 may be an improved estimate of the variance (standard deviation) obtained from the concurrent period, **it may not be an improvement over the entire short** record variance (standard deviation) in case 2. It is necessary to compare the variance of the adjusted variance (equation 7-8) to the **variance of the variance ($S_{y_3}{}^2$) for the entire period (N_3). Compute the variance of the short-record variance ($S_{y_3}{}^2$) using equation 7-11.**

$$\text{Var}\left(S_{y_3}{}^2\right) = \frac{2\left(S_{y_3}\right)^4}{N_3 - 1} \qquad (7\text{-}11)$$

*

*

where all terms are previously defined. If the variance of equation 7-11 is smaller than the variance of S_y^2 given in equation 7-8, then use S_{y_3} as the final estimate of the standard deviation. Otherwise, use the value of S_y determined from equation 7-10.

FURTHER CONSIDERATIONS

The above equations were developed under the assumption that the concurrent observations of flows at the short and long-term sites have a joint normal probability distribution with a skewness of zero. When this assumption is seriously violated, the above equations are not exact and this technique should be used with caution. In addition, the reliability of r depends on the length of the concurrent period, N_1. To obtain a reliable estimate of r, N_1 should be at least 10 years.

Notice that it is not necessary to estimate the actual annual peaks from the regression equation but only the adjusted logarithmic mean and standard deviation. The adjusted skew coefficient should be computed by weighting the generalized skew with the skew computed from the short record site as described in Section V.B.4.

*

*

NOTATION

N_1 = Number of years when flows were concurrently observed at the two sites

N_2 = Number of years when flows were observed at the longer record site but not observed at the short record site

N_3 = Number of years of flow at the short record site

N_e = Equivalent years of record of the adjusted mean

S_y = Standard deviation of the logarithm of flows for the extended period at the short record site

S_{x_1} = Standard deviation of logarithm of flows at the long record site during concurrent period

S_{x_2} = Standard deviation of logarithm of flows at the long record site for the period when flows were not observed at the short record site

S_{y_1} = Standard deviation of the logarithm of flows at the short record site for the concurrent period

S_{y_2} = not used

S_{y_3} = Standard deviation of logarithm of flows for the entire period at the short record site

X_1 = Logarithms of flows from long record during concurrent period

$\overline{X}_1$ = Mean logarithm of flows at the long record site for the concurrent period

$\overline{X}_2$ = Mean logarithm of flows at the long record site for the period when flow records are not available at the short record site

$\overline{X}_3$ = Mean logarithm of flows for the entire period at the long record site

Y_1 = Logarithms of flows from short record during concurrent period

$\overline{Y}$ = Mean logarithm of flows for the extended period at the short record site

$\overline{Y}_1$ = Mean logarithm of flows for the period of observed flow at the short record site (concurrent period)

$\overline{Y}_2$ = not used

*

*

$\overline{Y}_3$ = Mean logarithm of flows for the entire period at the short record site

b = Regression coefficient for Y_1 on X_1

r = Correlation coefficient of the flows at the two sites for concurrent periods

*

*

TABLE 7-1 MINIMUM r VALUES FOR IMPROVING MEAN OR STANDARD DEVIATION ESTIMATES

CONCURRENT RECORD	MEAN	STANDARD DEVIATION
10	0.35	0.65
11	0.33	0.62
12	0.32	0.59
13	0.30	0.57
14	0.29	0.55
15	0.28	0.54
16	0.27	0.52
17	0.26	0.50
18	0.25	0.49
19	0.24	0.48
20	0.24	0.47
21	0.23	0.46
22	0.22	0.45
23	0.22	0.44
24	0.21	0.43
25	0.21	0.42
26	0.20	0.41
27	0.20	0.41
28	0.20	0.40
29	0.19	0.39
30	0.19	0.39
31	0.19	0.38
32	0.18	0.37
33	0.18	0.37
34	0.18	0.36
35	0.17	0.36

*

Appendix 8

WEIGHTING OF INDEPENDENT ESTIMATES

The following procedure is suggested for adjusting flow frequency estimates based upon short records to reflect flood experience in nearby hydrologically similar watersheds, using any one of the various *generalization methods mentioned in V.C.1 . The procedure is based upon* the assumption that the estimates are independent, which for practical purposes is true in most situations.

If two independent estimates are weighted inversely proportional to their variance, the variance of the weighted average, z, is less than the variance of either estimate. According to Gilroy (30), if

$$z = \frac{x(V_y) + y(V_x)}{V_y + V_x} \tag{8-1}$$

then

$$V_z = \frac{V_x V_y}{(V_x + V_y)^2} \left[V_x + V_y + 2r\sqrt{V_x V_y}\right] \tag{8-2}$$

in which V_x, V_y, and V_z are the variances of x, y, and z respectively, and r is the cross correlation coefficient between values of x and values of y. Thus, if two estimates are independent, r is zero and

$$V_z = \frac{V_x V_y}{V_x + V_y} \tag{8-3}$$

As the variance of flood events at selected exceedance probabilities computed by the Pearson Type III procedure is inversely proportional to the number of annual events used to compute the statistics (25), equation (8-3) can be written

$$C/N_z = \frac{(C/N_x)\ (C/N_y)}{C/N_x + C/N_y} = \frac{C}{N_x + N_y} \tag{8-4}$$

in which C is a constant, N_x and N_y are the number of annual events used to compute x and y respectively, and N_z is the number of events that would be required to give a flood event at the selected exceedance probabilities with a variance equivalent to that of z computed by equation 8-1. Therefore,

$$N_z = N_x + N_y \tag{8-5}$$

From equation 8-1,

$$Z = \frac{xC/N_y + yC/N_x}{C/N_x + C/N_y} = \frac{x(N_x) + y(N_y)}{N_x + N_y} \tag{8-6}$$

Equation 8-6 can be used to weight independent estimates of the logarithms of flood discharges at selected probabilities and equation 8-5 can be used to appraise the accuracy of the weighted average. As a flood frequency discharge estimated by generalization tends to be independent of that obtained from the station data, such weighting is often justified particularly if the stations used in the generalization cover an area with a radius of over 100 miles or if their period of record is long in comparison with that at the station for which the estimate is being made. For generalizations based on stations covering a smaller area or with shorter records, the accuracy of the weighted average given by equation 8-6 is less than given by equation 8-5.

For cases where the estimates from the generalization and from the station data are not independent, the accuracy of the weighted estimate is reduced depending on the cross correlation of the estimates.

Given a peak discharge of 1,000 cfs with exceedance probability of 0.02 from a generalization with an accuracy equivalent to an estimate based on a 10-year record, for example, and an independent estimate of 2,000 cfs from 15 annual peaks observed at the site, the weighted average would be given by substitution in equation 8-6 as follows:

$$\text{Log } Q_{.02} = \frac{10(\log 1000) + 15(\log 2000)}{25} = 3.181$$

from which $Q_{.02}$ is 1,520 cfs. By equation 8-5 this estimate is as good as would be obtained from 25 annual peaks.

If an expected probability adjustment is to be applied to a weighted estimate, the adjustment to probability should be the same as that applicable to samples from normal distributions as described in Appendix 11, but N should be that for a sample size that gives equivalent accuracy. Thus, in the preceding example, the expected probability adjustment would be that for a sample of size 25 taken from a normal distribution.

*

Appendix 9

CONFIDENCE LIMITS

The record of annual peak flows at a site is a random sample of the underlying population of annual peaks and can be used to estimate the frequency curve of that population. If the same size random sample could be selected from a different period of time, a different estimate of the underlying population frequency curve probably would result. Thus, an estimated flood frequency curve can be only an approximation to the true frequency curve of the underlying population of annual flood peaks. To gauge the accuracy of this approximation, one may construct an interval or range of hypothetical frequency curves that, with a high degree of confidence, contains the population frequency curve. Such intervals are called confidence intervals and their end points are called confidence limits.

This appendix explains how to construct confidence intervals for flood discharges that have specified exceedance probabilities. To this end, let X_P^* denote the true or population logarithmic discharge that has exceedance probability P. Upper and lower confidence limits for X_P^* , with confidence level c, are defined to be numbers $U_{P,c}(X)$ and $L_{P,c}(X)$, based on the observed flood records, X, such that the upper confidence limit $U_{P,c}(X)$ lies above X_P^* with probability c and the lower limit $L_{P,c}(X)$ lies below X_P^* with probability c. That is, the confidence limits have the property that

$$\text{Probability} \quad \{U_{P,c}(X) \geq X_P^*\} = c \qquad (9\text{-}1a)$$

$$\text{Probability} \quad \{L_{P,c}(X) \leq X_P^*\} = c \qquad (9\text{-}1b)$$

*

*Explicit formulas for computing the confidence limits are given below; the above formulas simply explain the statistical meaning of the confidence limits.

The confidence limits defined above are called <u>one-sided</u> confidence limits because each of them describes a bound or limit on just one side of the population p-probability discharge. A two-sided confidence interval can be formed from the overlap or union of the two one-sided intervals, as follows:

$$\text{Probability} \left\{ L_{P,c}(X) \leq X_P^* \leq U_{P,c}(X) \right\} = 2c-1 \qquad (9\text{-}2)$$

Thus, the union of two one-sided 95-percent confidence intervals is a two-sided 90-percent interval. It should be noted that the two-sided interval so formed may not be the narrowest possible interval with that confidence level; nevertheless, it is considered satisfactory for use with these guidelines.

It may be noted in the above equations that $U_{P,c}(X)$ can lie above X_P^* if and only if $U_{P,c}(X)$ lies above a fraction (1-P) of all possible floods in the population. In quality control terminology, $U_{P,c}(X)$ would be called an upper <u>tolerance limit</u>, at confidence level c, for the proportion (1-P) of the population. Similarly, $L_{P,c}(X)$ would be a lower tolerance limit for the proportion (P). Because the tolerance limit terminology refers to proportions of the population, whereas the confidence-limit terminology refers directly to the discharge of interest, the confidence-limit terminology is adopted in these guidelines.

Explicit formulas for the confidence limits are derived by specifying the general form of the limits and making additional simplifying assumptions to analyze the relationships between sample statistics and population statistics. The general form of the confidence limits is specified as:

$$U_{P,c}(X) = \overline{X} + S\left(K^U_{P,c}\right) \qquad (9\text{-}3a)$$

$$L_{P,c}(X) = \overline{X} + S\left(K^L_{P,c}\right) \qquad (9\text{-}3b)$$

*

*

in which $\overline{X}$ and S are the logarithmic mean and standard deviation of the final estimated log Pearson Type III frequency curve and $K^U_{P,c}$ and $K^L_{P,c}$ are upper and lower confidence coefficients.

The confidence coefficients approximate the non-central t-distribution. The non-central t-variate can be obtained in tables (41, 32), although the process is cumbersome when G_w is non-zero. More convenient is the use of the following approximate formulas (32, pp. 2-15), based on a large sample approximation to the non-central t-distribution (42):

$$K^U_{P,c} = \frac{K_{G_w,P} + \sqrt{K^2_{G_w,P} - ab}}{a} \quad (9\text{-}4a)$$

$$K^L_{P,c} = \frac{K_{G_w,P} - \sqrt{K^2_{G_w,P} - ab}}{a} \quad (9\text{-}4b)$$

in which

$$a = 1 - \frac{z_c^2}{2(N-1)} \quad (9\text{-}5)$$

$$b = K^2_{G_w,P} - \frac{z_c^2}{N} \quad (9\text{-}6)$$

and z_c is the standard normal deviate (zero-skew Pearson Type III deviate) with cumulative probability c (exceedance probability 1-c). The systematic record length N is deemed to control the statistical reliability of the estimated frequency curve and is to be used for calculating confidence limits even when historic information has been used to estimate the frequency curve.

The use of equations 9-3 through 9-6 is illustrated by calculating 95-percent confidence limits for $X^*_{0.01}$, the 0.01 exceedance probability flood, when the estimated frequency curve has logarithmic mean, standard deviation, and skewness of 3.00, 0.25, and 0.20, respectively based on 50 years of systematic record.

*

*

$z_c = 1.645$ ✓ $\qquad K_{G_W,P} = 2.4723$ ✓

$z_{.05}$

$$a = 1 - \frac{(1.645)^2}{98} = 0.9724$$

$$b = (2.4723)^2 - \frac{(1.645)^2}{50} = 6.058$$

$$K^u_{0.01,\,0.95} = \frac{2.4723 + \sqrt{(2.4723)^2 - (0.9724)(6.058)}}{0.9724}$$

$$= 3.026$$

$$K^L_{0.01,\,0.95} = \frac{2.4723 - \sqrt{(2.4723)^2 - (0.9724)(6.058)}}{0.9724}$$

$$= 2.059$$

$$U_{0.01,\,0.95}(X) = 3.00 + (0.25)(3.026) = 3.756$$

$$L_{0.01,\,0.95}(X) = 3.00 + (0.25)(2.059) = 3.515$$

The corresponding limits in natural units (cubic feet per second) are 3270 and 5700; the estimated 0.01 exceedance probability flood is 4150 cubic feet per second.

Table 9-1 is a portion of the non-central t tables (43) for a skew of zero and can be used to compute $K^U_{P,c}$ and $K^L_{P,c}$ for selected values of P and c when the distribution of logarithms of the annual peaks is normal (i.e., G_W=0).

An example of using table 9-1 to compute confidence limits is as follows: Assume the 95-percent confidence limits are desired for $X^*_{0.01}$, the 0.01 exceedance probability flood for a frequency curve with logarithmic mean, standard deviation and skewness of 3.00, 0.25 and 0.00, respectively, based on 50 years of systematic record.

*

*$K^{U}_{0.01,\,0.95} = 2.862$ Found by entering table 9-1 with confidence level 0.05, systematic record length 50 and exceedance probability 0.01.

$K^{L}_{0.01,\,0.95} = 1.936$ Found by entering table 9-1 with confidence level 0.95, systematic record length 50 and exceedance probability 0.01.

$$U_{0.01,\,0.95}(X) = 3.00 + 0.25(2.862) = 3.715$$

$$L_{0.01,\,0.95}(X) = 3.00 + 0.25(1.936) = 3.484$$

The corresponding limits in natural units (cubic feet per second) are 3050 and 5190; the estimated 0.01 exceedance probability flood is 3820 cubic feet per second. *

*

Appendix 9 Notation

$U_{P,c}(X)$ = upper confidence limit in log units

$L_{P,c}(X)$ = lower confidence limit in log units

P = exceedance probability

c = confidence level

X_P^* = population logarithmic discharge for exceedance probability P

$\overline{X}$ = mean logarithm of peak flows

S = standard deviation of logarithms of annual peak discharges

$K_{G_w,P}$ = Pearson Type III coordinate expressed in number of standard deviations from the mean for weighted skew (G_w) and exceedance probability (P).

G_w = weighted skew coefficient

$K_{P,c}^U$ = upper confidence coefficient

$K_{P,c}^L$ = lower confidence coefficient

N = systematic record length

z_c = is the standard normal deviate *

*

TABLE 9-1
CONFIDENCE LIMIT DEVIATE VALUES FOR NORMAL DISTRIBUTION

Confidence Level	Systematic Record Length	EXCEEDANCE PROBABILITY											
	N	.002	.005	.010	.020	.040	.100	.200	.500	.800	.900	.950	.990
.01	10	6.178	5.572	5.074	4.535	3.942	3.048	2.243	.892	-.107	-.508	-.804	-1.314
	15	5.147	4.639	4.222	3.770	3.274	2.521	1.841	.678	-.236	-.629	-.929	-1.458
	20	4.675	4.212	3.832	3.419	2.965	2.276	1.651	.568	-.313	-.705	-1.008	-1.550
	25	4.398	3.960	3.601	3.211	2.782	2.129	1.536	.498	-.364	-.757	-1.064	-1.616
	30	4.212	3.792	3.447	3.071	2.658	2.030	1.457	.450	-.403	-.797	-1.107	-1.667
	40	3.975	3.577	3.249	2.893	2.500	1.902	1.355	.384	-.457	-.854	-1.169	-1.741
	50	3.826	3.442	3.125	2.781	2.401	1.821	1.290	.340	-.496	-.894	-1.212	-1.793
	60	3.723	3.347	3.038	2.702	2.331	1.764	1.244	.309	-.524	-.924	-1.245	-1.833
	70	3.647	3.278	2.974	2.644	2.280	1.722	1.210	.285	-.545	-.948	-1.272	-1.865
	80	3.587	3.223	2.924	2.599	2.239	1.688	1.183	.265	-.563	-.968	-1.293	-1.891
	90	3.538	3.179	2.883	2.561	2.206	1.661	1.160	.250	-.578	-.984	-1.311	-1.913
	100	3.498	3.143	2.850	2.531	2.179	1.639	1.142	.236	-.591	-.998	-1.326	-1.932
.05	10	4.862	4.379	3.981	3.549	3.075	2.355	1.702	.580	-.317	-.712	-1.017	-1.563
	15	4.304	3.874	3.520	3.136	2.713	2.068	1.482	.455	-.406	-.802	-1.114	-1.677
	20	4.033	3.628	3.295	2.934	2.534	1.926	1.370	.387	-.460	-.858	-1.175	-1.749
	25	3.868	3.478	3.158	2.809	2.425	1.838	1.301	.342	-.497	-.898	-1.217	-1.801
	30	3.755	3.376	3.064	2.724	2.350	1.777	1.252	.310	-.525	-.928	-1.250	-1.840
	40	3.608	3.242	2.941	2.613	2.251	1.697	1.188	.266	-.565	-.970	-1.297	-1.896
	50	3.515	3.157	2.862	2.542	2.188	1.646	1.146	.237	-.592	-1.000	-1.329	-1.936
	60	3.448	3.096	2.807	2.492	2.143	1.609	1.116	.216	-.612	-1.022	-1.354	-1.966
	70	3.399	3.051	2.765	2.454	2.110	1.581	1.093	.199	-.629	-1.040	-1.374	-1.990
	80	3.360	3.016	2.733	2.425	2.083	1.559	1.076	.186	-.642	-1.054	-1.390	-2.010
	90	3.328	2.987	2.706	2.400	2.062	1.542	1.061	.175	-.652	-1.066	-1.403	-2.026
	100	3.301	2.963	2.684	2.380	2.044	1.527	1.049	.166	-.662	-1.077	-1.414	-2.040

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TABLE 9-1 (CONTINUED)
CONFIDENCE LIMIT DEVIATE VALUES FOR NORMAL DISTRIBUTION

EXCEEDANCE PROBABILITY

Confidence Level	Systematic Record Length N	.002	.005	.010	.020	.040	.100	.200	.500	.800	.900	.950	.990
.10	10	4.324	3.889	3.532	3.144	2.716	2.066	1.474	.437	-.429	-.828	-1.144	-1.715
	15	3.936	3.539	3.212	2.857	2.465	1.867	1.320	.347	-.499	-.901	-1.222	-1.808
	20	3.743	3.364	3.052	2.712	2.338	1.765	1.240	.297	-.541	-.946	-1.271	-1.867
	25	3.623	3.255	2.952	2.623	2.258	1.702	1.190	.264	-.570	-.978	-1.306	-1.908
	30	3.541	3.181	2.884	2.561	2.204	1.657	1.154	.239	-.593	-1.002	-1.332	-1.940
	40	3.433	3.082	2.793	2.479	2.131	1.598	1.106	.206	-.624	-1.036	-1.369	-1.986
	50	3.363	3.019	2.735	2.426	2.084	1.559	1.075	.184	-.645	-1.059	-1.396	-2.018
	60	3.313	2.974	2.694	2.389	2.051	1.532	1.052	.167	-.662	-1.077	-1.415	-2.042
	70	3.276	2.940	2.662	2.360	2.025	1.511	1.035	.155	-.674	-1.091	-1.431	-2.061
	80	3.247	2.913	2.638	2.338	2.006	1.495	1.021	.144	-.684	-1.103	-1.444	-2.077
	90	3.223	2.891	2.618	2.319	1.989	1.481	1.010	.136	-.693	-1.112	-1.454	-2.090
	100	3.203	2.873	2.601	2.305	1.976	1.470	1.001	.129	-.701	-1.120	-1.463	-2.101
.25	10	3.599	3.231	2.927	2.596	2.231	1.671	1.155	.222	-.625	-1.043	-1.382	-2.008
	15	3.415	3.064	2.775	2.460	2.112	1.577	1.083	.179	-.661	-1.081	-1.422	-2.055
	20	3.320	2.978	2.697	2.390	2.050	1.528	1.045	.154	-.683	-1.104	-1.448	-2.085
	25	3.261	2.925	2.648	2.346	2.011	1.497	1.020	.137	-.699	-1.121	-1.466	-2.106
	30	3.220	2.888	2.614	2.315	1.984	1.475	1.002	.125	-.710	-1.133	-1.479	-2.123
	40	3.165	2.838	2.568	2.274	1.948	1.445	.978	.108	-.726	-1.151	-1.499	-2.147
	50	3.129	2.805	2.538	2.247	1.924	1.425	.962	.096	-.738	-1.164	-1.513	-2.163
	60	3.105	2.783	2.517	2.227	1.907	1.411	.950	.088	-.747	-1.173	-1.523	-2.176
	70	3.085	2.765	2.501	2.213	1.893	1.401	.942	.081	-.753	-1.181	-1.532	-2.186
	80	3.070	2.752	2.489	2.202	1.883	1.392	.935	.076	-.759	-1.187	-1.538	-2.194
	90	3.058	2.740	2.478	2.192	1.875	1.386	.929	.071	-.763	-1.192	-1.544	-2.201
	100	3.048	2.731	2.470	2.184	1.868	1.380	.925	.068	-.767	-1.196	-1.549	-2.207

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TABLE 9-1 (CONTINUED)
CONFIDENCE LIMIT DEVIATE VALUES FOR NORMAL DISTRIBUTION

Confidence Level	Systematic Record Length N	EXCEEDANCE PROBABILITY .002	.005	.010	.020	.040	.100	.200	.500	.800	.900	.950	.990
.75	10	2.508	2.235	2.008	1.759	1.480	1.043	.625	-.222	-1.155	-1.671	-2.104	-2.927
	15	2.562	2.284	2.055	1.803	1.521	1.081	.661	-.179	-1.083	-1.577	-1.991	-2.775
	20	2.597	2.317	2.085	1.831	1.547	1.104	.683	-.154	-1.045	-1.528	-1.932	-2.697
	25	2.621	2.339	2.106	1.851	1.566	1.121	.699	-.137	-1.020	-1.497	-1.895	-2.648
	30	2.641	2.357	2.123	1.867	1.580	1.133	.710	-.125	-1.002	-1.475	-1.869	-2.614
	40	2.668	2.383	2.147	1.888	1.600	1.151	.726	-.108	-.978	-1.445	-1.834	-2.568
	50	2.688	2.400	2.163	1.903	1.614	1.164	.738	-.096	-.962	-1.425	-1.811	-2.538
	60	2.702	2.414	2.176	1.916	1.625	1.173	.747	-.088	-.950	-1.411	-1.795	-2.517
	70	2.714	2.425	2.186	1.925	1.634	1.181	.753	-.081	-.942	-1.401	-1.782	-2.501
	80	2.724	2.434	2.194	1.932	1.640	1.187	.759	-.076	-.935	-1.392	-1.772	-2.489
	90	2.731	2.441	2.201	1.938	1.646	1.192	.763	-.071	-.929	-1.386	-1.764	-2.478
	100	2.739	2.447	2.207	1.944	1.652	1.196	.767	-.068	-.925	-1.380	-1.758	-2.470
.90	10	2.165	1.919	1.715	1.489	1.234	.828	.429	-.437	-1.474	-2.066	-2.568	-3.532
	15	2.273	2.019	1.808	1.576	1.314	.901	.499	-.347	-1.320	-1.867	-2.329	-3.212
	20	2.342	2.082	1.867	1.630	1.364	.946	.541	-.297	-1.240	-1.765	-2.208	-3.052
	25	2.390	2.126	1.908	1.669	1.400	.978	.570	-.264	-1.190	-1.702	-2.132	-2.952
	30	2.426	2.160	1.940	1.698	1.427	1.002	.593	-.239	-1.154	-1.657	-2.080	-2.884
	40	2.479	2.209	1.986	1.740	1.465	1.036	.624	-.206	-1.106	-1.598	-2.010	-2.793
	50	2.517	2.244	2.018	1.770	1.493	1.059	.645	-.184	-1.075	-1.559	-1.965	-2.735
	60	2.544	2.269	2.042	1.792	1.513	1.077	.662	-.167	-1.052	-1.532	-1.933	-2.694
	70	2.567	2.290	2.061	1.810	1.529	1.091	.674	-.155	-1.035	-1.511	-1.909	-2.662
	80	2.585	2.307	2.077	1.824	1.543	1.103	.684	-.144	-1.021	-1.495	-1.890	-2.638
	90	2.600	2.321	2.090	1.836	1.553	1.112	.693	-.136	-1.010	-1.481	-1.874	-2.618
	100	2.613	2.333	2.101	1.847	1.563	1.120	.701	-.129	-1.001	-1.470	-1.861	-2.601

TABLE 9-1 (CONTINUED)
CONFIDENCE LIMIT DEVIATE VALUES FOR NORMAL DISTRIBUTION

Confidence Level	Systematic Record Length N	EXCEEDANCE PROBABILITY .002	.005	.010	.020	.040	.100	.200	.500	.800	.900	.950	.990
.95	10	1.989	1.757	1.563	1.348	1.104	.712	.317	-.580	-1.702	-2.355	-2.911	-3.981
	15	2.121	1.878	1.677	1.454	1.203	.802	.406	-.455	-1.482	-2.068	-2.566	-3.520
	20	2.204	1.955	1.749	1.522	1.266	.858	.460	-.387	-1.370	-1.926	-2.396	-3.295
	25	2.264	2.011	1.801	1.569	1.309	.898	.497	-.342	-1.301	-1.838	-2.292	-3.158
	30	2.310	2.053	1.840	1.605	1.342	.928	.525	-.310	-1.252	-1.777	-2.220	-3.064
	40	2.375	2.113	1.896	1.657	1.391	.970	.565	-.266	-1.188	-1.697	-2.125	-2.941
	50	2.421	2.156	1.936	1.694	1.424	1.000	.592	-.237	-1.146	-1.646	-2.065	-2.862
	60	2.456	2.188	1.966	1.722	1.450	1.022	.612	-.216	-1.116	-1.609	-2.022	-2.807
	70	2.484	2.214	1.990	1.745	1.470	1.040	.629	-.199	-1.093	-1.581	-1.990	-2.765
	80	2.507	2.235	2.010	1.762	1.487	1.054	.642	-.186	-1.076	-1.559	-1.964	-2.733
	90	2.526	2.252	2.026	1.778	1.500	1.066	.652	-.175	-1.061	-1.542	-1.944	-2.706
	100	2.542	2.267	2.040	1.791	1.512	1.077	.662	-.166	-1.049	-1.527	-1.927	-2.684
.99	10	1.704	1.492	1.314	1.115	.886	.508	.107	-.892	-2.243	-3.048	-3.738	-5.074
	15	1.868	1.645	1.458	1.251	1.014	.629	.236	-.678	-1.841	-2.521	-3.102	-4.222
	20	1.974	1.743	1.550	1.336	1.094	.705	.313	-.568	-1.651	-2.276	-2.808	-3.832
	25	2.050	1.813	1.616	1.399	1.152	.757	.364	-.498	-1.536	-2.129	-2.633	-3.601
	30	2.109	1.867	1.667	1.446	1.196	.797	.403	-.450	-1.457	-2.030	-2.515	-3.447
	40	2.194	1.946	1.741	1.515	1.259	.854	.457	-.384	-1.355	-1.902	-2.364	-3.249
	50	2.255	2.002	1.793	1.563	1.304	.894	.496	-.340	-1.290	-1.821	-2.269	-3.125
	60	2.301	2.045	1.833	1.600	1.337	.924	.524	-.309	-1.244	-1.764	-2.202	-3.038
	70	2.338	2.079	1.865	1.630	1.365	.948	.545	-.285	-1.210	-1.722	-2.153	-2.974
	80	2.368	2.107	1.891	1.653	1.387	.968	.563	-.265	-1.183	-1.688	-2.114	-2.924
	90	2.394	2.131	1.913	1.674	1.405	.984	.578	-.250	-1.160	-1.661	-2.082	-2.883
	100	2.416	2.151	1.932	1.691	1.421	.998	.591	-.236	-1.142	-1.639	-2.056	-2.850

Appendix 10

RISK

This appendix describes the recommended procedures for estimating the risk incurred when a location is occupied for a period of years. As used in this guide, risk is defined as the probability that one or more events will exceed a given flood magnitude within a specified period of years.

Two basic approaches may be used to compute risk, nonparametric methods [(e.g., (19)] and parametric methods [(e.g., (20)]. Parametric methods which use the binomial distribution require assuming that the annual exceedance frequency is exactly known. The difference between methods is not great, particularly in the range of usual interest; consequently, use of the binomial distribution is recommended because of ease of comprehension and application.

The binomial expression for estimating risk is:

$$R_I = \frac{N!}{I!\,(N-I)!} P^I \, (1-P)^{N-I} \qquad (10\text{-}1)$$

in which R_I is the estimated risk of obtaining in N years exactly I number of flood events exceeding a flood magnitude with annual exceedance probability P.

When I equals 0 equation 10-1 reduces to:

$$R_o = (1-P)^N \qquad (10\text{-}2)$$

in which R_o is the estimated probability of nonexceedance of the selected flood magnitude in N years. From this the risk R of one or more exceedance becomes

$$R\ (1 \text{ or more}) = 1 - (1-P)^N \qquad (10\text{-}3)$$

Risk of 2 or more exceedances, R (2 or more), is

$$R(2 \text{ or more}) = R-R_1 = R-NP\,(1-P)^{N-1} \qquad (10\text{-}4)$$

* Some solutions are illustrated by the following table and figure * 10-1.

*

BINOMIAL RISK TABLE

TIME	** RISK (PERCENT) ** P=0.100 NONE	ONE OR MORE	TWO OR MORE	** RISK (PERCENT) ** P=0.050 NONE	ONE OR MORE	TWO OR MORE
10	35	65	26	60	40	9
20	12	88	61	36	64	26
30	4	96	82	21	79	45
40	1	99	92	13	87	60
50	1	99	97	8	92	72
60	0	100	99	5	95	81
70	0	100	99	3	97	87
80	0	100	100	2	98	91
90	0	100	100	1	99	94
100	0	100	100	1	99	96
110	0	100	100	0	100	98
120	0	100	100	0	100	98
150	0	100	100	0	100	100
200	0	100	100	0	100	100

TIME	** RISK (PERCENT) ** P=0.040 NONE	ONE OR MORE	TWO OR MORE	** RISK (PERCENT) ** P=0.020 NONE	ONE OR MORE	TWO OR MORE
10	66	34	6	82	18	2
20	44	56	19	67	33	6
30	29	71	34	55	45	12
40	20	80	48	45	55	19
50	13	87	60	36	64	26
60	9	91	70	30	70	34
70	6	94	78	24	76	41
80	4	96	83	20	80	48
90	3	97	88	16	84	54
100	2	98	91	13	87	60
110	1	99	94	11	89	65
120	1	99	96	9	91	69
150	0	100	98	5	95	80
200	0	100	100	2	98	91

NOTE: TABLE VALUES ARE ROUNDED TO NEAREST PERCENT

*

*

BINOMIAL RISK TABLE

TIME	** RISK (PERCENT) ** P=0.010 NONE	ONE OR MORE	TWO OR MORE	** RISK (PERCENT) ** P=0.005 NONE	ONE OR MORE	TWO OR MORE
10	90	10	0	95	5	0
20	82	18	2	90	10	0
30	74	26	4	86	14	1
40	67	33	6	82	18	2
50	61	39	9	78	22	3
60	55	45	12	74	26	4
70	49	51	16	70	30	5
80	45	55	19	67	33	6
90	40	60	23	64	36	8
100	37	63	26	61	39	9
110	33	67	30	58	42	11
120	30	70	34	55	45	12
150	22	78	44	47	53	17
200	13	87	60	37	63	26

TIME	** RISK (PERCENT) ** P=0.002 NONE	ONE OR MORE	TWO OR MORE	** RISK (PERCENT) ** P=0.001 NONE	ONE OR MORE	TWO OR MORE
10	98	2	0	99	1	0
20	96	4	0	98	2	0
30	94	6	0	97	3	0
40	92	8	0	96	4	0
50	90	10	0	95	5	0
60	89	11	1	94	6	0
70	87	13	1	93	7	0
80	85	15	1	92	8	0
90	84	16	1	91	9	0
100	82	18	2	90	10	0
110	80	20	2	90	10	1
120	79	21	2	89	11	1
150	74	26	4	86	14	1
200	67	33	6	82	18	2

NOTE: TABLE VALUES ARE ROUNDED TO NEAREST PERCENT

*

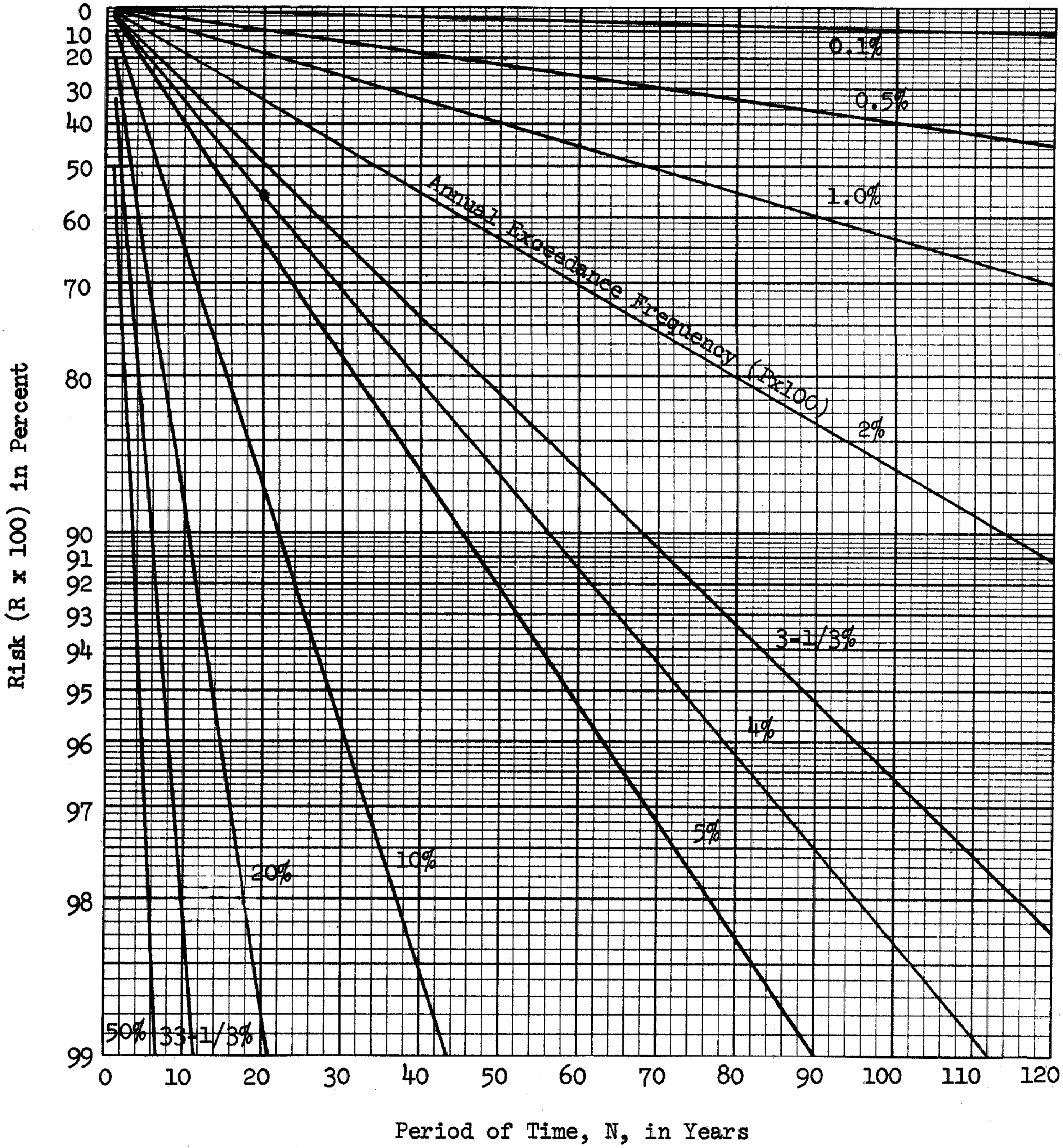


Figure 10-1. RISK OF ONE OR MORE FLOOD EVENTS EXCEEDING A FLOOD OF GIVEN ANNUAL EXCEEDANCE FREQUENCY WITHIN A PERIOD OF YEARS

Appendix 11
EXPECTED PROBABILITY

The principle of gambling based upon estimated probabilities can be applied to water resources development decisions. However, because probabilities must be inferred from random sample data, they are uncertain and mathematical expectation cannot be computed exactly as errors due to uncertainty do not necessarily compensate. For example, if the estimate based on sample data is that a certain flood magnitude will be exceeded on the average once in 100 years, it is possible that the true exceedance could be three or four more times per hundred years, but it can never be less than zero times per hundred years. The impact of errors in one direction due to uncertainty can be quite different from the impact of errors in the other direction. Thus, it is not adequate to simply be too high half the time and too low the other half. It is necessary to consider the relative impacts of being too high or too low.

It is possible to delineate uncertainty with considerable accuracy when dealing with samples from a normal distribution. Therefore, when flood flow frequency curves conform fairly closely to the logarithmic normal distribution, it is possible to delineate uncertainty of frequency or probability estimates of flood flows.

Figure 11-1 is a generalized representation of the range of uncertainty in probability estimates based on samples drawn from a normal population. The vertical scale can represent the logarithm of streamflow. The curves show the likelihood that the true frequency of any flood magnitude exceeds the value shown on the frequency scale. The curve labeled .50 is the curve that would be used for the best frequency estimate of a log-normal population. From this curve a magnitude of 2 would be exceeded on the average 30 times per thousand events. The figure also shows a 5 percent chance that the true frequency is 150 or more times per thousand or a 5 percent chance that the true frequency is two times or less per thousand events.

If a magnitude of 2.0 were selected at 20 independent locations, the best estimate for the frequency is 3 exceedances per hundred years for each location. The estimated total exceedance for all 20 locations would be 60 per 100 years. However, due to sampling uncertainties, true frequencies for a magnitude of 2.0 would differ at each location and total exceedances per 100 years at the 20 locations might be represented by the following tabulation.

Exceedances Per 100 Years at Each of 20 Locations*

20	5	3	.9	
12	5	2	.8	
10	4	2	.5	Total Exceedances = Approximately 90
8	4	2	.3	
7	3	1	.1	

*Determined from Figure 11-1 using 0.05 parameter value increments from .025 through .975.

The total of these exceedances is about 90 per 100 years or 30 more than obtained using the best probability estimate as the true probability at each location. If, however, the mathematically derived expected probability function were used instead of the traditional "best" estimate we could read the expected probability curve of Figure 11-1 to obtain the value of about 4.5 exceedances per 100 events. This value when applied to each of the 20 locations would give an estimate of 90 exceedances per 100 years at all 20 locations. Thus, while the expected probability estimate would be wrong in the high direction more frequently than in the low direction, the heavier impacts of being wrong in the low direction would compensate for this. It can be noted, at this point, that expected probability is the average of all estimated true probabilities.

If a flood frequency estimate could be accurately known--that is, the parent population could be defined--the frequency distribution of observed flood events would approach the parent population as the number of observations approaches infinity. This is not the case where probabilities are not accurately known. However, if the expected probabilities as illustrated in Figure 11-1 can be computed, observed flood frequency for a large number of independent locations will approach the estimated flood frequency as the number of observations approaches infinity and the number of locations approaches infinity.

It appears that the answer to the question as to whether expected probability should be used at a single location would be identical to the answer to the question, "What is a fair wager for a single gamble?" If the gamble must be undertaken, and ordinarily it must, then the answer to the above question is that the wager should be proportional to the expected return. In determining whether the expected probability concepts should apply for a single location, the same line of reasoning would indicate that it should.

It has been shown (21) that for the normal distribution the expected probability P_N can be obtained from the formula

$$P_N = \text{Prob}\left[t_{N-1} > K_n \left(\frac{N}{N+1}\right)^{1/2}\right] \qquad (11\text{-}1)$$

where K_n is the standard normal variate of the desired probability of exceedance, N is the sample size, and t_{N-1} is the Student's t-statistic with N-1 degrees of freedom.

The actual calculations can be carried out using tables of the t-statistic, or the modified values shown in Table 11-1 (31). To use Table 11-1, enter with the sample size minus 1 and read across to the column with the desired exceedance probability. The value read from the table is the corrected plotting position.

The expected probability correction may also be calculated from the following equations (34) which are based on Table 11-1. For selected exceedance probabilities greater than 0.50, and a given sample size, the appropriate P_N value equals 1 minus the value in Table 11-1 or the equations 11-2.

Exceedance Probability	Expected Probability, P_N	
.0001	$.0001\ (1.0 + 1600/N^{1.72})$	(11-2a)
.001	$.001\ (1.0 + 280/N^{1.55})$	(11-2b)
.01	$.01\ (1.0 + 26/N^{1.16})$	(11-2c)
.05	$.05\ (1.0 + 6/N^{1.04})$	(11-2d)
.10	$.1\ (1.0 + 3/N^{1.04})$	(11-2e)
.30	$.3\ (1.0 + 0.46/N^{0.925})$	(11-2f)

For floods with an exceedance probability of 0.01 based on samples of 20 annual peaks, for example, the expected probability of exceedance from equation 11-2c is (.01) (1.0 + 26/32.3) or 0.018. Use of Table 11-1 gives 0.0174. Comparable equations for adjusting the computed discharge upward to give a discharge for which the expected probability equals the exceedance probability are available (22).

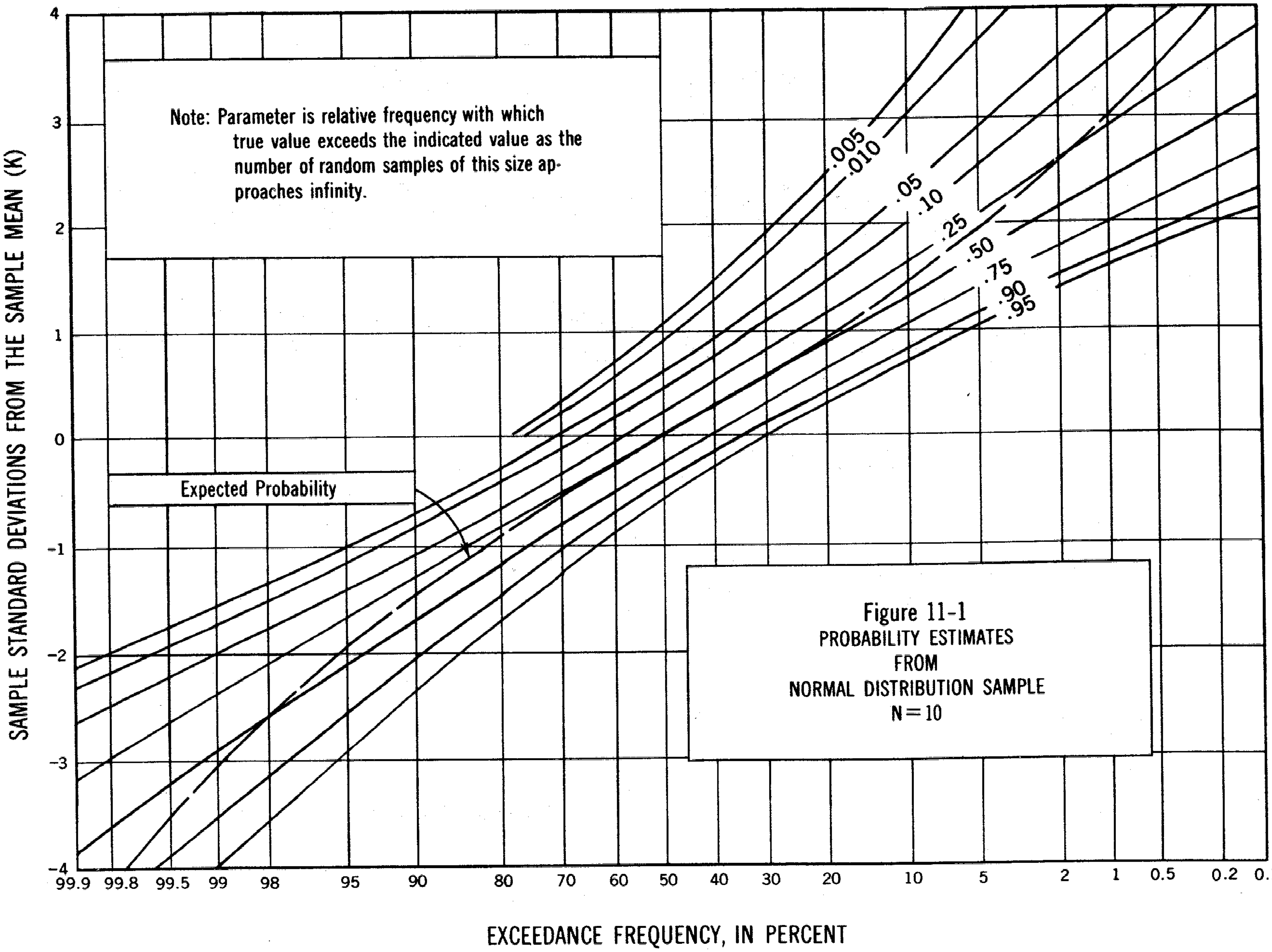


Figure 11-1
PROBABILITY ESTIMATES
FROM
NORMAL DISTRIBUTION SAMPLE
N=10

Table 11-1

TABLE OF P_N VERSUS P_∞

For use with samples drawn from a normal population

N-1 \ P_∞	.50	.30	.10	.05	.01	.001	.0001
1	.500	.372	.243	.204	.154	.121	.102
2	.500	.347	.193	.146	.090	.057	.043
3	.500	.336	.169	.119	.064	.035	.023
4	.500	.330	.154	.104	.050	.024	.0137
5	.500	.325	.146	.094	.042	.0179	.0092
6	.500	.322	.138	.088	.036	.0138	.0066
7	.500	.319	.135	.083	.032	.0113	.0050
8	.500	.317	.131	.079	.029	.0094	.0039
9	.500	.316	.127	.076	.027	.0082	.0031
10	.500	.315	.125	.073	.025	.0072	.0025
11	.500	.314	.123	.071	.023	.0064	.0021
12	.500	.313	.121	.069	.022	.0058	.0018
13	.500	.312	.119	.068	.021	.0052	.0016
14	.500	.311	.118	.067	.020	.0048	.0014
15	.500	.311	.117	.066	.0196	.0045	.0013
16	.500	.310	.116	.065	.0190	.0042	.0012
17	.500	.310	.115	.064	.0184	.0040	.0011
18	.500	.309	.114	.063	.0179	.0038	.0010
19	.500	.309	.113	.062	.0174	.0036	.00091
20	.500	.308	.113	.062	.0170	.0034	.00084
21	.500	.308	.112	.061	.0167	.0033	.00078
22	.500	.308	.111	.061	.0163	.0031	.00073
23	.500	.307	.111	.060	.0161	.0030	.00068
24	.500	.307	.110	.060	.0158	.0029	.00064
25	.500	.307	.110	.059	.0155	.0028	.00060
26	.500	.306	.109	.059	.0153	.0027	.00057
27	.500	.306	.109	.059	.0151	.0026	.00054
28	.500	.306	.109	.058	.0149	.0026	.00051
29	.500	.306	.108	.058	.0147	.0025	.00049
30	.500	.306	.108	.058	.0145	.0024	.00046
40	.500	.304	.106	.056	.0133	.0020	.00034
60	.500	.303	.104	.054	.0122	.0016	.00025
120	.500	.302	.102	.052	.0111	.0013	.00017
∞	.500	.300	.100	.050	.0100	.0010	.00010

NOTE: P_N values above are usable approximately with Pearson Type III distributions having small skew coefficients.

*

Appendix 12

FLOW DIAGRAM AND EXAMPLE PROBLEMS *

The sequence of procedures recommended by this guide for defining flood potentials (except for the case of mixed populations) is described in the following outline and flow diagrams.

A. Determine available data and data to be used.
   1. Previous studies
   2. Gage records
   3. Historic data
   4. Studies for similar watersheds
   5. Watershed model

B. Evaluate data.
   1. Record homogeneity
   2. Reliability and accuracy

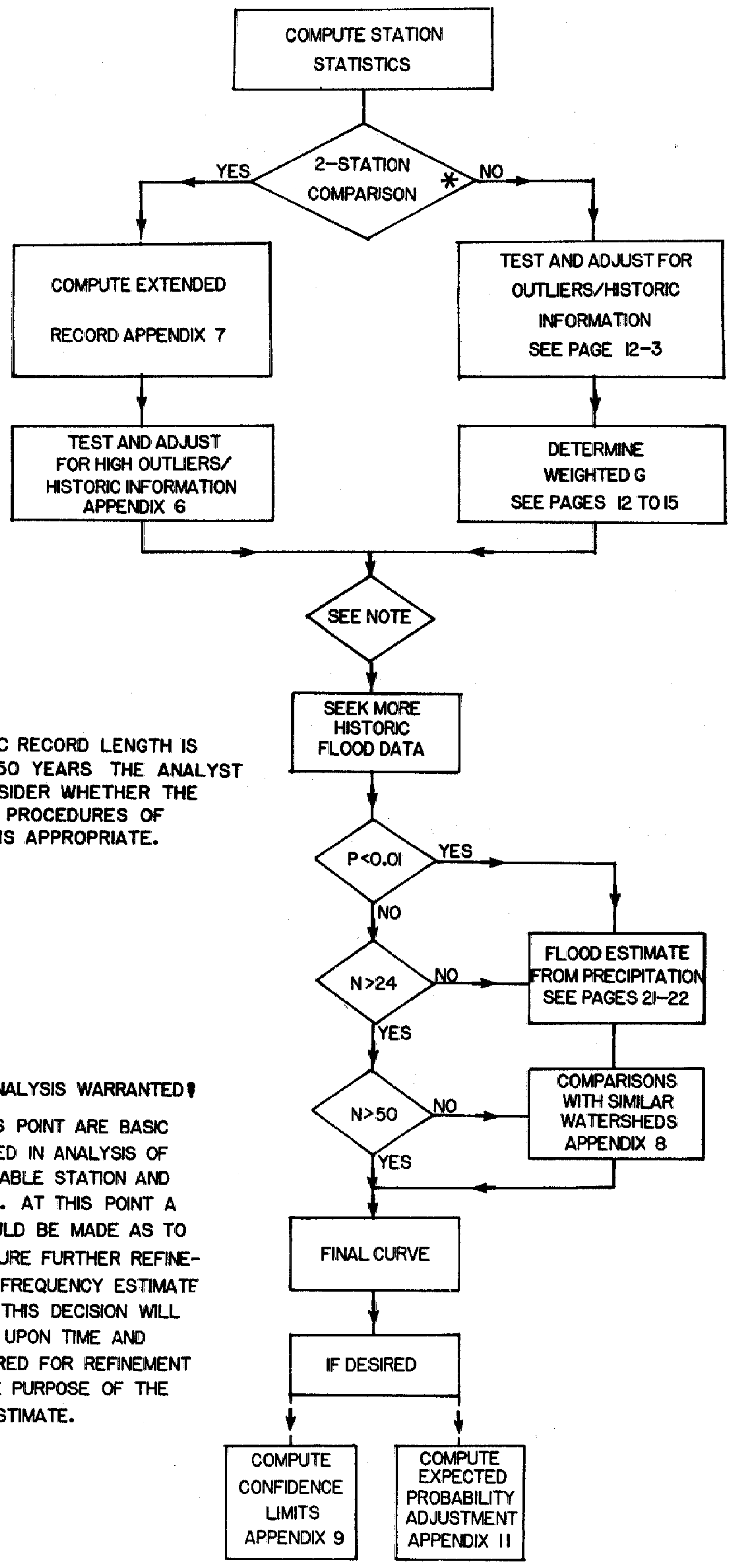
c. Compute curve following guide procedures as outlined in following flow diagrams. Example problems showing most of the computational techniques follow the flow diagram.

*

ZERO FLOOD
OR
INCOMPLETE RECORD

COMPLETE RECORD

SEE APPENDIX 5, CONDITIONAL PROBABILITY ADJUSTMENT, FOR OUTLIERS SEE PAGES 17 TO 19 AND APPENDIX 5 AND 6



* IF SYSTEMATIC RECORD LENGTH IS LESS THAN 50 YEARS THE ANALYST SHOULD CONSIDER WHETHER THE USE OF THE PROCEDURES OF APPENDIX 7 IS APPROPRIATE.

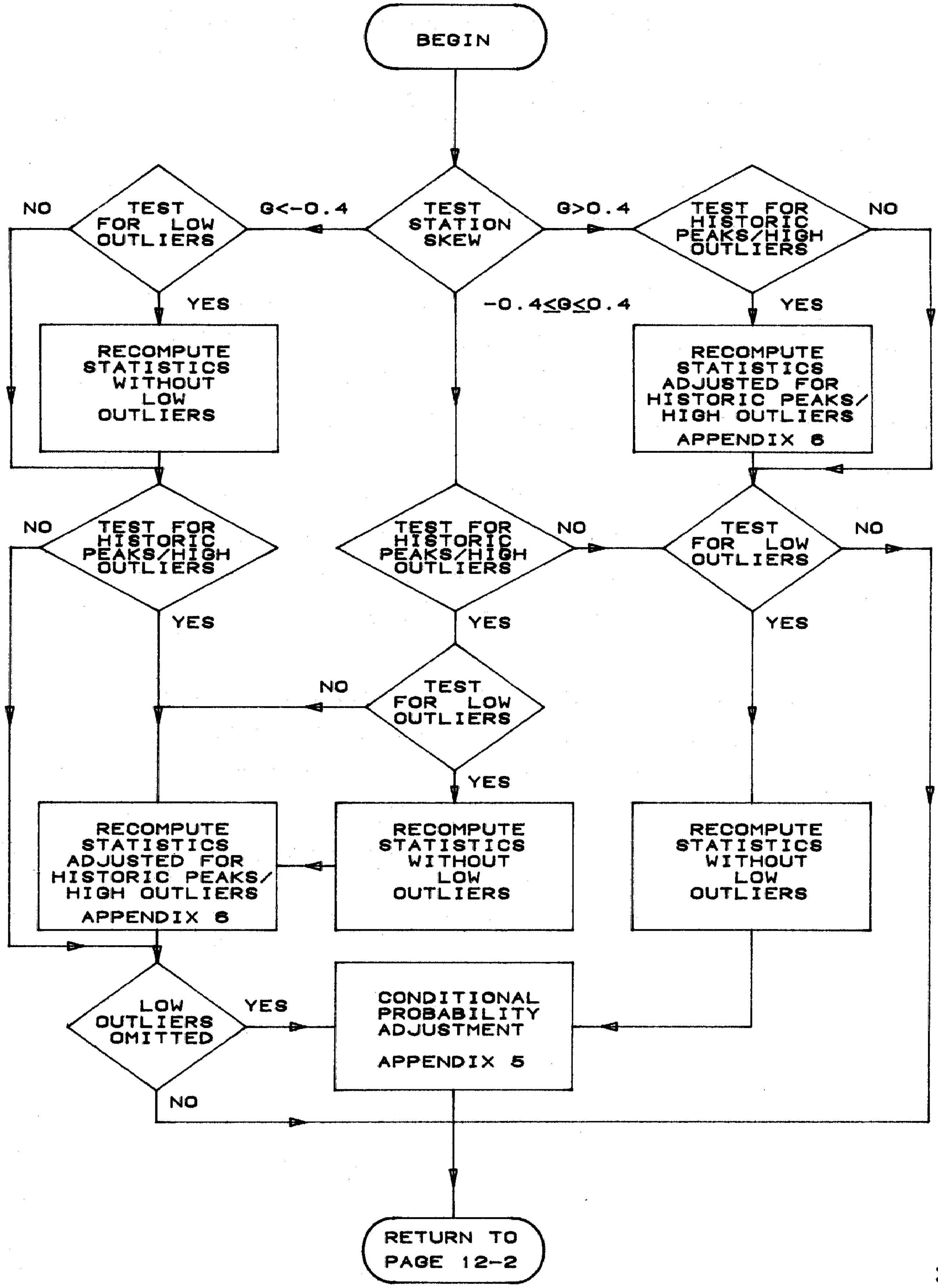
NOTE: IS FURTHER ANALYSIS WARRANTED?

STEPS TO THIS POINT ARE BASIC STEPS REQUIRED IN ANALYSIS OF READILY AVAILABLE STATION AND HISTORIC DATA. AT THIS POINT A DECISION SHOULD BE MADE AS TO WHETHER FUTURE FURTHER REFINEMENT OF THE FREQUENCY ESTIMATE IS JUSTIFIED. THIS DECISION WILL DEPEND BOTH UPON TIME AND EFFORT REQUIRED FOR REFINEMENT AND UPON THE PURPOSE OF THE FREQUENCY ESTIMATE.

FLOW DIAGRAM FOR FLOOD FLOW FREQUENCY ANALYSIS

*

* FLOW DIAGRAM FOR HISTORIC AND OUTLIER ADJUSTMENT



*

The following examples illustrate application of most of the techniques recommended in this guide. Annual flood peak data for four stations (Table 12-1) have been selected to illustrate the following:

1. Fitting the Log-Pearson Type III distribution
2. Adjusting for high outliers
3. Testing and adjusting for low outliers
4. Adjusting for zero flood years

The procedure for adjusting for historic flood data is given in Appendix 6 and an example computation is provided. An example has not been included specifically for the analysis of an incomplete record as this technique is applied in Example 4, adjusting for zero flood years. The computation of confidence limits and the adjustment for expected probability are described in Example 1. The generalized *skew coefficient used in these examples was taken from Plate I. In actual practice, the generalized skew may be obtained from other sources or a special study made for the region. *

Because of round off errors in the computational procedures, computed values may differ beyond the second decimal point.

* These examples have been completely revised using the procedures recommended in Bulletin 17B. Specific changes have not been indicated on the following pages: *

TABLE 12-1

ANNUAL FLOOD PEAKS FOR FOUR STATIONS IN EXAMPLES

Year	Fishkill Creek 01-3735 Example 1	Floyd River 06-6005 Example 2	Back Creek 01-6140 Example 3	Orestimba Creek 11-2745 Example 4
1929			8750	
1930			15500	
1931			4060	
1932				4260
1933				345
1934				516
1935		1460		1320
1936		4050	22000*	1200
1937		3570	-	2180
1938		2060	-	3230
1939		1300	6300	115
1940		1390	3130	3440
1941		1720	4160	3070
1942		6280	6700	1880
1943		1360	22400	6450
1944		7440	3880	1290
1945	2290	5320	8050	5970
1946	1470	1400	4020	782
1947	2220	3240	1600	0
1948	2970	2710	4460	0
1949	3020	4520	4230	335
1950	1210	4840	3010	175
1951	2490	8320	9150	2920
1952	3170	13900	5100	3660
1953	3220	71500	9820	147
1954	1760	6250	6200	0
1955	8800	2260	10700	16
1956	8280	318	3880	5620
1957	1310	1330	3420	1440
1958	2500	970	3240	10200
1959	1960	1920	6800	5380
1960	2140	15100	3740	448
1961	4340	2870	4700	0
1962	3060	20600	4380	1740
1963	1780	3810	5190	8300
1964	1380	726	3960	156
1965	980	7500	5600	560
1966	1040	7170	4670	128
1967	1580	2000	7080	4200
1968	3630	829	4640	0
1969		17300	536	5080
1970		4740	6680	1010
1971		13400	8360	584
1972		2940	18700	0
1973		5660	5210	1510

*Not included in example computations.

EXAMPLE 1

FITTING THE LOG-PEARSON TYPE III DISTRIBUTION

a. <u>Station Description</u>

Fishkill Creek at Beacon, New York

USGS Gaging Station: 01-3735
Lat: 41°30'42", long: 73°56'58"
Drainage Area: 190 sq. mi.
Annual Peaks Available: 1945-1968

b. <u>Computational Procedures</u>

Step 1 - List data, transform to logarithms, and compute the squares and the cubes.

TABLE 12-2
COMPUTATION OF SUMMATIONS

Year	Annual Peak (cfs)	Logarithm (X)	X^2	X^3
1945	2290	3.35984	11.28852	37.92764
1946	1470	3.16732	10.03192	31.77429
1947	2220	3.34635	11.19806	37.47262
1948	2970	3.47276	12.06006	41.88170
1949	3020	3.48001	12.11047	42.14456
1950	1210	3.08279	9.50359	29.29759
1951	2490	3.39620	11.53417	39.17236
1952	3170	3.50106	12.25742	42.91397
1953	3220	3.50786	12.30508	43.16450
1954	1760	3.24551	10.53334	34.18604
1955	8800	3.94448	15.55892	61.37186
1956	8280	3.91803	15.35096	60.14552
1957	1310	3.11727	9.71737	30.29167
1958	2500	3.39794	11.54600	39.23260
1959	1960	3.29226	10.83898	35.68473
1960	2140	3.33041	11.09163	36.93968
1961	4340	3.63749	13.23133	48.12884
1962	3060	3.48572	12.15024	42.35235
1963	1780	3.25042	10.56523	34.34144
1964	1380	3.13988	9.85885	30.95559
1965	980	2.99123	8.94746	26.76390
1966	1040	3.01703	9.10247	27.46243
1967	1580	3.19866	10.23143	32.72685
1968	3630	3.55991	12.67296	45.11459
N=24	--	Σ 80.84043	273.68646	931.44732

Step 2 - Computation of mean by Equation 2:

$$\bar{X} = \frac{\Sigma X}{N}$$

$$= \frac{80.84043}{24} = 3.3684 \qquad (12\text{-}1)$$

Computation of standard deviation by Equation 3b:

$$S = \left[\frac{\Sigma X^2 - (\Sigma X)^2/N}{N-1}\right]^{0.5}$$

$$S = \left[\frac{273.68646 - (80.84043)^2/24}{23}\right]^{0.5} \qquad (12\text{-}2)$$

$$S = \sqrt{\frac{1.38750}{23}} = 0.2456$$

Computation of skew coefficient by Equation 4b:

$$G = \frac{N^2(\Sigma X^3) - 3N(\Sigma X)(\Sigma X^2) + 2\,(\Sigma X)^3}{N(N-1)(N-2)S^3}$$

$$= \frac{(24)^2(931.44732) - 3(24)(80.84043)(273.68646) + 2(80.84043)^3}{24(24-1)\qquad(24-2)\qquad(.24561)^3}$$

$$= \frac{536513.6563 - 1592995.0400 + 1056612.7341}{(24)(23)(22)\qquad(.014816)} \qquad (12\text{-}3)$$

$$= \frac{131.3504}{179.9285} = 0.7300$$

Example 1 - Fitting the Log-Pearson Type III Distribution (continued)

Step 3 - Check for Outliers:

$$X_H = \overline{X} + K_N S$$

$$= 3.3684 + 2.467\ (.2456) \quad = 3.9743 \qquad (12\text{-}4)$$

$$Q_H = \text{antilog}\ (3.9743) \quad = \quad 9425 \text{ cfs}$$

The largest recorded value does not exceed the threshold value. Next, the test for detecting possible low outliers is applied. The same K_N value is used in equation 8a to compute the low outlier threshold (Q_L):

$$X_L = \overline{X} - K_N S$$

$$= 3.3684 - 2.467(.2456) \quad = 2.7625 \qquad (12\text{-}5)$$

$$Q_L = \text{antilog}\ (2.7625) \quad = \quad 579 \text{ cfs}$$

There are no recorded values below this threshold value. No outliers were detected by either the high or low tests. For this example a generalized skew of 0.6 is determined from Plate I. In actual practice a generalized skew may be obtained from other sources or from a special study made for the region. A weighted skew is computed by use of Equation 5. The mean square error of the station skew can be found within Table 1 or computed by Equation 6. Computation of mean-square error of station skew by Eq. 6:

$$MSE_G \simeq 10^{\left[A - B\ [\log_{10}(N/10)]\right]}$$

Where:

$$A = -0.33 + 0.08\ |G| = -0.33 + 0.08(.730) \quad = -.2716 \qquad (12\text{-}6)$$

$$B = 0.94 - 0.26\ |G| \quad = \quad 0.94 - 0.26(.730) = \quad .7502 \qquad (12\text{-}7)$$

$$MSE_G \simeq 10^{\left[-.2716 - .7502\ [\log_{10}(2.4)]\right]} \simeq 10^{-.55683} \simeq 0.277 \qquad (12\text{-}8)$$

The mean-square error of the generalized skew from Plate I is 0.302.

Computation of weighted skew by equation 5:

$$G_W = \frac{MSE_{\overline{G}}\,(G) + MSE_G\,(\overline{G})}{MSE_{\overline{G}} + MSE_G}$$

$$= \frac{.302(.7300) + .277(.6)}{.579} = 0.6678 \qquad (12\text{-}9)$$

$$= 0.7 \text{ (rounded to nearest tenth)}$$

Step 4 - Compute the frequency curve coordinates.

The log-Pearson Type III K values for a skew coefficient of 0.7 are found in Appendix 3. An example computation for an exceedance probability of .01 using Equation 1 follows:

$$\log Q = \overline{X} + KS = 3.3684 + 2.82359(.2456) = 4.0619 \qquad (12\text{-}10)$$

$$Q = 11500 \text{ cfs}$$

The discharge values in this computation and those in Table 12-3 are rounded to three significant figures.

TABLE 12-3

COMPUTATION OF FREQUENCY CURVE COORDINATES

P	$K_{G_W,P}$ for $G_W = 0.7$	log Q	Q cfs
.99	-1.80621	2.9247	841
.90	-1.18347	3.0777	1200
.50	-0.11578	3.3399	2190
.10	1.33294	3.6957	4960
.05	1.81864	3.8150	6530
.02	2.40670	3.9595	9110
.01	2.82359	4.0619	11500
.005	3.22281	4.1599	14500
.002	3.72957	4.2844	19200

The frequency curve is plotted in Figure 12-1.

Step 5 - Compute the confidence limits.

The upper and lower confidence limits for levels of significance of .05 and .95 percent are computed by the procedures outlined in Appendix 9. Nine exceedance probabilities (P) have been selected to define the confidence limit curves. The computations for two points on the curve at an exceedance probability of 0.99 are given below.

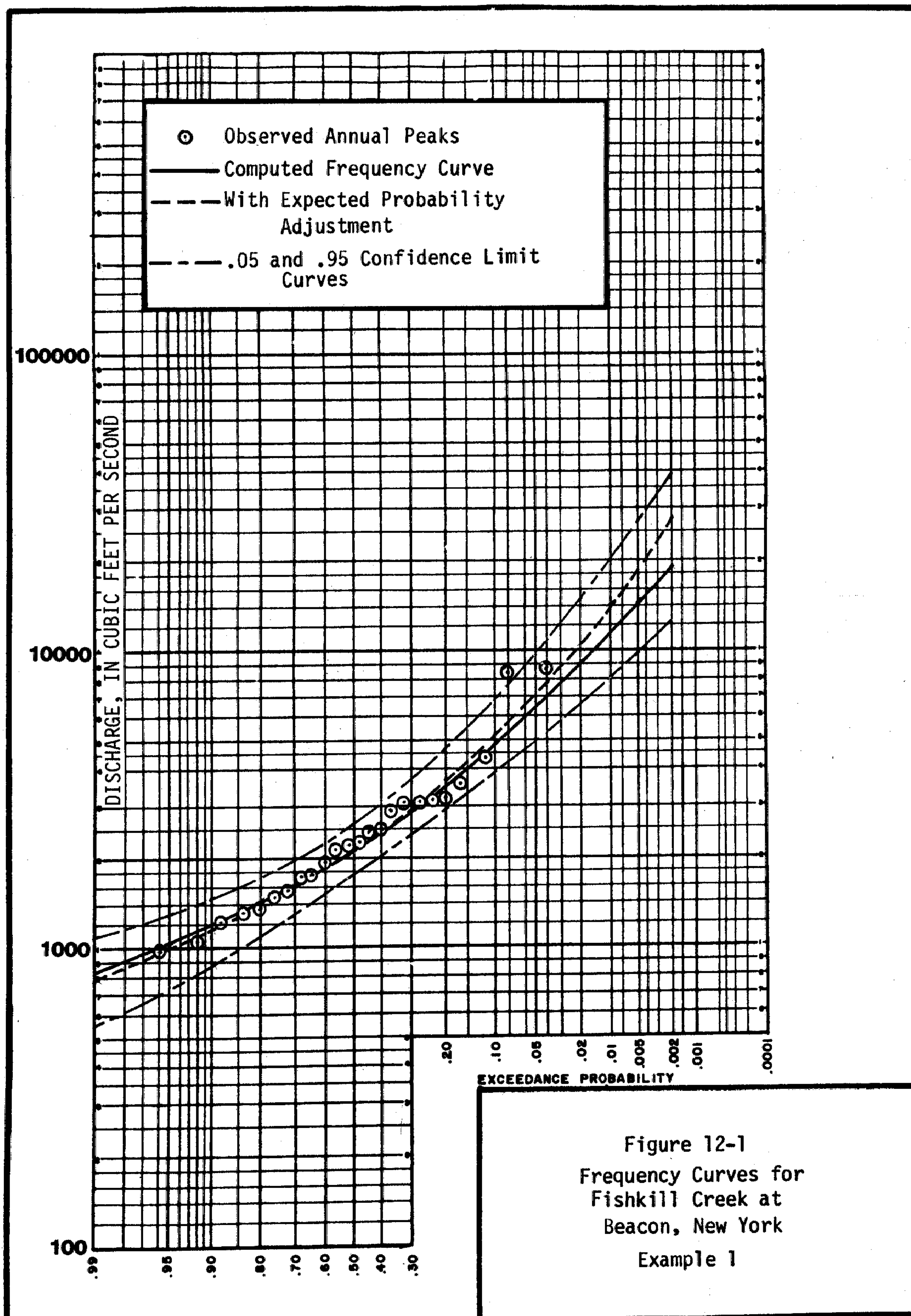


Figure 12-1
Frequency Curves for
Fishkill Creek at
Beacon, New York

Example 1

Example 1 - Fitting the Log-Pearson Type III Distribution (continued)

Equations in Appendix 9 are used in computing an approximate value for $K_{P,c}$. The normal deviate, z_c, is found by entering Appendix 3 with a skew coefficient of zero. For a confidence level of 0.05, $z_c = 1.64485$. The Pearson Type III deviates, $K_{G_w,P}$ are found in Appendix 3 based on the appropriate skew coefficient. For an exceedance probability of 0.99 and skew coefficient of 0.7, $K_{G_w,P} = -1.80621$.

$$a = 1-\frac{z_c^2}{2(N-1)} = 1 - \frac{(1.64485)^2}{2(24-1)} = 0.9412 \tag{12-11}$$

$$b = K^2_{G_w,P} - \frac{z_c^2}{N} = (-1.80621)^2 - \frac{(1.64485)^2}{24} = 3.1497 \tag{12-12}$$

$$K^U_{P,c} = \frac{K_{G_w,P}+\sqrt{K^2_{G_w,P} - ab}}{a} = \frac{-1.80621 + \sqrt{(-1.80621)^2-(.9412)(3.1497)}}{.9412} \tag{12-13}$$

$$= \frac{-1.80621 + .5458}{.9412} = -1.3392$$

The discharge value is:

$$\text{Log } Q = 3.3684 + (-1.3392)(.2456) \tag{12-14}$$

$$= 3.0395$$

$$Q = 1100$$

For the lower confidence coefficient:

$$K^L_{P,c} = \frac{K_{G_w,P}-\sqrt{K^2_{G_w,P} - ab}}{a} = \frac{-1.80621 - .5458}{.9412} = -2.4989 \tag{12-15}$$

Example 1 - Fitting the Log-Pearson Type III Distribution (continued)

The discharge value is:

$$\text{Log } Q = 3.3684 + (-2.4989)(.2456) \qquad (12\text{-}16)$$

$$= 2.7546$$

$$Q = 568$$

The computations showing the derivation of the upper and lower confidence limits are given in Table 12-4. The resulting curves are shown in Figure 12-1.

TABLE 12-4
COMPUTATION OF CONFIDENCE LIMITS

P	$K_{G_W,P}$ for $G_W = 0.7$	0.05 UPPER LIMIT CURVE $K^U_{P,c}$	log Q	Q cfs	0.05 LOWER LIMIT CURVE $K^L_{P,c}$	log Q	Q cfs
.99	-1.80621	-1.3392	3.0395	1100	-2.4989	2.7546	568
.90	-1.18347	-0.7962	3.1728	1490	-1.7187	2.9462	884
.50	-0.11578	0.2244	3.4235	2650	-0.4704	3.2528	1790
.10	1.33294	1.9038	3.8359	6850	0.9286	3.5964	3950
.05	1.81864	2.5149	3.9860	9680	1.3497	3.6998	5010
.02	2.40670	3.2673	4.1708	14800	1.8469	3.8220	6640
.01	2.82359	3.8058	4.3031	20100	2.1943	3.9073	8080
.005	3.22281	4.3239	4.4303	26900	2.5245	3.9884	9740
.002	3.72957	4.9841	4.5925	39100	2.9412	4.0907	12300

Step 6 - Compute the expected probability adjustment.

The expected probability plotting positions are determined from Table 11-1 based on N - 1 of 23.

TABLE 12-5

EXPECTED PROBABILITY ADJUSTMENT

P	Q	Expected Probability
.99	841	.9839
.90	1200	.889
.50	2190	.50
.10	4960	.111
.05	6530	.060
.02	9110	.028*
.01	11500	.0161
.005	14500	.0095*
.002	19200	.0049*

*Interpolated values

The frequency curve adjusted for expected probability is shown in Figure 12-1.

EXAMPLE 2

ADJUSTING FOR A HIGH OUTLIER

a. Station Description

Floyd River at James, Iowa

USGS Gaging Station: 06-6005
Lat: 42°34'30", long: 96° 18'45"
Drainage Area: 882 sq. mi.
Annual Peaks Available: 1935-1973

b. Computational Procedures

Step 1 - Compute the statistics.

The detailed computations for the systematic record 1935-1973 have been omitted; the results of the computations are:

Mean Logarithm	3.5553
Standard Deviation of logs	0.4642
Skew Coefficient of logs	0.3566
Years	39

At this point, the analyst may wish to see the preliminary frequency curve based on the statistics of the systematic record. Figure 12-2 is the preliminary frequency curve based on the computed mean and standard deviation and a weighted skew of 0.1 (based on a generalized skew of -0.3 from Plate I).

Step 2 - Check for Outliers.

The station skew is between $\pm$ 0.4; therefore, the tests for both high outliers and low outliers are based on the systematic record statistics before any adjustments are made. From Appendix 4, the K_N for a sample size of 39 is 2.671.

The high outlier threshold (Q_H) is computed by Equation 7:

$$X_H = \overline{X} + K_N S$$

$$= 3.5553 + 2.671(.4642) = 4.7952 \qquad (12\text{-}17)$$

$$Q_H = \text{antilog } (4.7952) \quad = 62400 \text{ cfs}$$

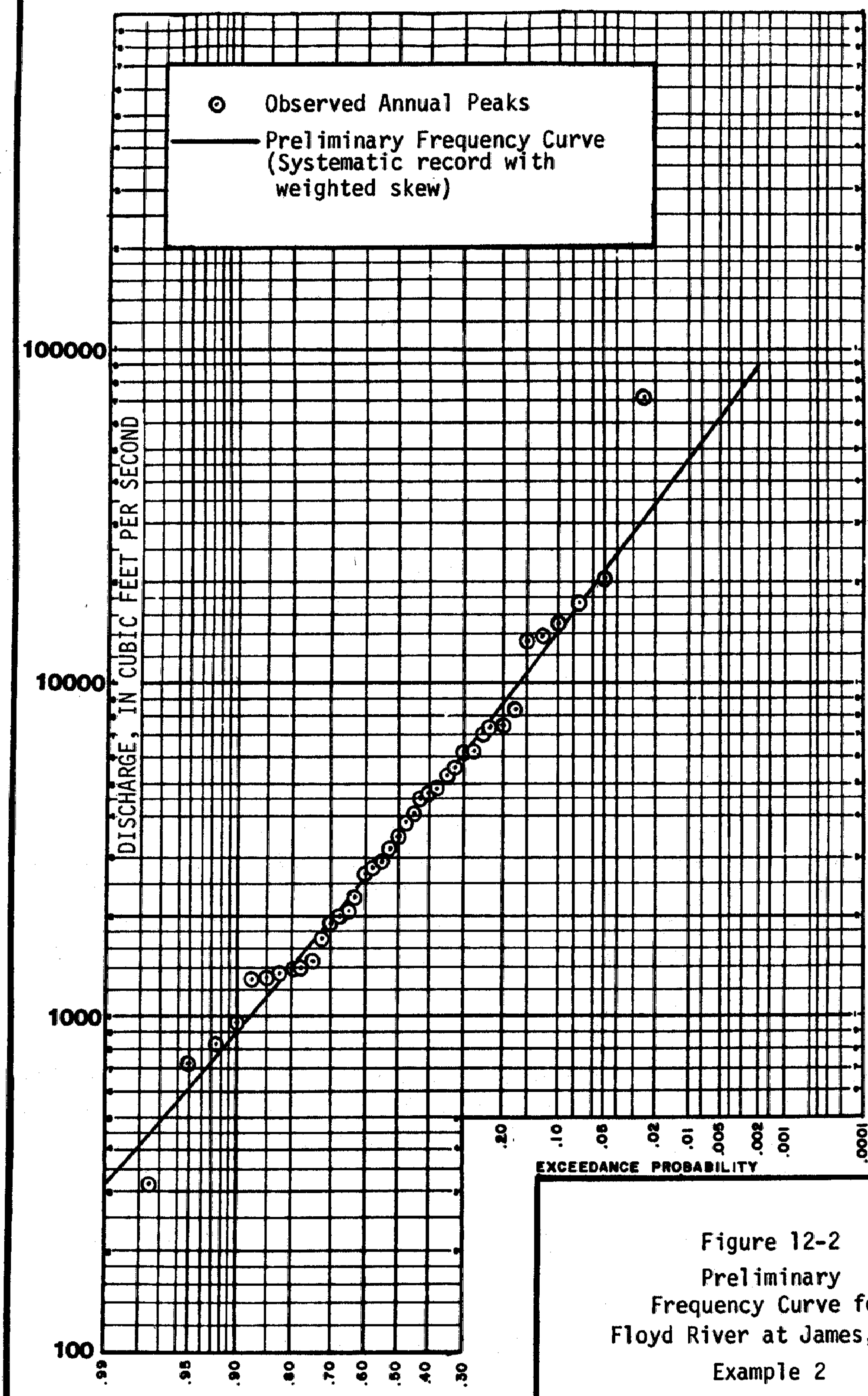


Figure 12-2
Preliminary
Frequency Curve for
Floyd River at James, Iowa
Example 2

The 1953 value of 71500 exceeds this value. Information from local residents indicates that the 1953 event is known to be the largest event since 1892; therefore, this event will be treated as a high outlier. If such information was not available, comparisons with nearby stations may have been desirable.

The low-outlier threshold (Q_L) is computed by Equation 8a:

$$X_L = \overline{X} - K_N S$$

$$= 3.5553 - 2.671(.4642) = 2.3154 \qquad (12\text{-}18)$$

$$Q_L = \text{antilog } (2.3154) = 207 \text{ cfs}$$

There are no values below this threshold value.

Step 3 - Recompute the statistics.

The 1953 value is deleted and the statistics recomputed from the remaining systematic record:

Mean Logarithm	3.5212
Standard Deviation of logs	0.4177
Skew Coefficient of logs	-0.0949
Years	38

Step 4 - Use historic data to modify statistics and plotting positions.

Application of the procedures in Appendix 6 allows the computed statistics to be adjusted by incorporation of the historic data.

(1) The historic period (H) is 1892-1973 or 82 years and the number of low values excluded (L) is zero.

(2) The systematic period (N) is 1935-1973 (with 1953 deleted) or 38 years.

(3) There is one event (Z) known to be the largest in 82 years.

(4) Compute weighting factor (W) by Equation 6-1:

$$W = \frac{H - Z}{N + L}$$

$$= \frac{82 - 1}{38 + 0} = 2.13158 \qquad (12\text{-}19)$$

Example 2 - Adjusting for a High Outlier (continued)

Compute adjusted mean by Equation 6-2b:

$$\tilde{M} = \frac{WNM + \Sigma X_z}{H-WL}$$

$$\bar{X} = M = 3.5212$$

$$WNM = 285.2173$$

$$\Sigma X_z = \underline{4.8543}$$

$$290.0716$$

$$\tilde{M} = 290.0716/(82-0) = 3.5375 \qquad (12\text{-}20)$$

Compute adjusted standard deviation by Equation 6-3b:

$$\tilde{S}^2 = \frac{W(N-1)S^2 + WN(M-\tilde{M})^2 + \Sigma(X_z - \tilde{M})^2}{H-WL-1}$$

$$S = .4177$$

$$W(N-1)S^2 = 13.7604$$

$$WN(M-\tilde{M})^2 = .0215$$

$$\Sigma(X_z-\tilde{M})^2 = \underline{1.7340}$$

$$15.5159$$

$$\tilde{S}^2 = \frac{15.5159}{82-0-1} = .1916 \qquad (12\text{-}21)$$

$$\tilde{S} = .4377$$

Compute adjusted skew:

First compute adjusted skew on basis of record by Equation 6-4b:

Example 2 - Adjusting for a High Outlier (continued)

$$\tilde{G} = \frac{H - WL}{(H-WL-1)(H-WL-2)\tilde{S}^3}\left[\frac{W(N-1)(N-2)S^3G}{N} + 3W(N-1)(M-\tilde{M})S^2 + WN(M-\tilde{M})^3 + \Sigma(X_z - \tilde{M})^3\right]$$

$$G = -0.0949$$

$$\frac{W(N-1)(N-2)S^3G}{N} = -.5168$$

$$3W(N-1)(M-\tilde{M})S^2 = -.6729$$

$$WN(M-\tilde{M})^3 = -.0004$$

$$\Sigma(X_z-\tilde{M})^3 = \frac{2.2833}{1.0932}$$

$$\frac{H}{(H-WL-1)(H-WL-2)\tilde{S}^3} = .1509 \qquad (12\text{-}22)$$

$$G = .1509\ (1.0932) = .1650$$

Next compute weighted skew:

For this example, a generalized skew of -0.3 is determined from Plate I. Plate I has a stated mean-square error of 0.302. Interpolating in Table I, the mean-square error of the station skew, based on H of 82 years, is 0.073. The weighted skew is computed by use of Equation 5:

$$G_W = \frac{.302(.1650) + .073(-.3)}{.302 + .073} = 0.0745 \qquad (12\text{-}23)$$

G_W = 0.1 (rounded to nearest tenth)

Example 2 - Adjusting for High Outlier (continued)

Step 5 - Compute adjusted plotting positions for historic data.

For the largest event (Equation 6-6):

$\tilde{m}_1 = 1$

For the succeeding events (Equation 6-7):

$\tilde{m} = W\,E - (W-1)(Z + 0.5)$

$$\tilde{m}_2 = 2.1316(2) - (2.1316-1)(1 + .5) \qquad (12\text{-}24)$$

$= 2.5658$

For the Weibull Distribution a = 0; therefore, by Equation 6-8

$$\widetilde{PP} = \frac{\tilde{m}}{H + 1}(100)$$

$$\widetilde{PP}_1 = \frac{1}{82 + 1}(100) = 1.20 \qquad (12\text{-}25)$$

$$\widetilde{PP}_2 = \frac{2.5658}{83}(100) = 3.09 \qquad (12\text{-}26)$$

Exceedance probabilities are computed by dividing values obtained from Equation 12-26 by 100.

$$\frac{3.09}{100} = .0309$$

TABLE 12-6

COMPUTATION OF PLOTTING POSITIONS

					Weibull Plotting Position	
			Event Number	Weighted Order	Percent Chance	Exceedance Probability
Year	Q	W	E	m	$\widetilde{PP}$	$\widetilde{PP}$
1953	71500	1.0000	1	1.0000	1.20	.0120
1962	20600	2.1316	2	2.5658	3.09	.0309
1969	17300	2.1316	3	4.6974	5.66	.0566
1960	15100	2.1316	4	6.8290	8.23	.0823
1952	13900	2.1316	5	8.9606	10.80	.1080
1971	13400	2.1316	6	11.0922	13.36	.1336
1951	8320	2.1316	7	13.2238	15.93	.1593
1965	7500	2.1316	8	15.3554	18.50	.1850
1944	7440	2.1316	9	17.4870	21.07	.2107
1966	7170	2.1316	10	19.6186	23.64	.2364

Only the first 10 values are shown for this example

Example 2 - Adjusting for a High Outlier (continued)

Step 6 - Compute the frequency curve.

TABLE 12-7

COMPUTATION OF FREQUENCY CURVE COORDINATES

P	$K_{G_W,P}$ for $G_W = 0.1$	log Q	Q cfs
.99	-2.25258	2.5515	356
.90	-1.27037	2.9815	958
.50	-0.01662	3.5302	3390
.10	1.29178	4.1029	12700
.05	1.67279	4.2697	18600
.02	2.10697	4.4597	28800
.01	2.39961	4.5878	38700
.005	2.66965	4.7060	50800
.002	2.99978	4.8504	70900

The final frequency curve is plotted on Figure 12-3.

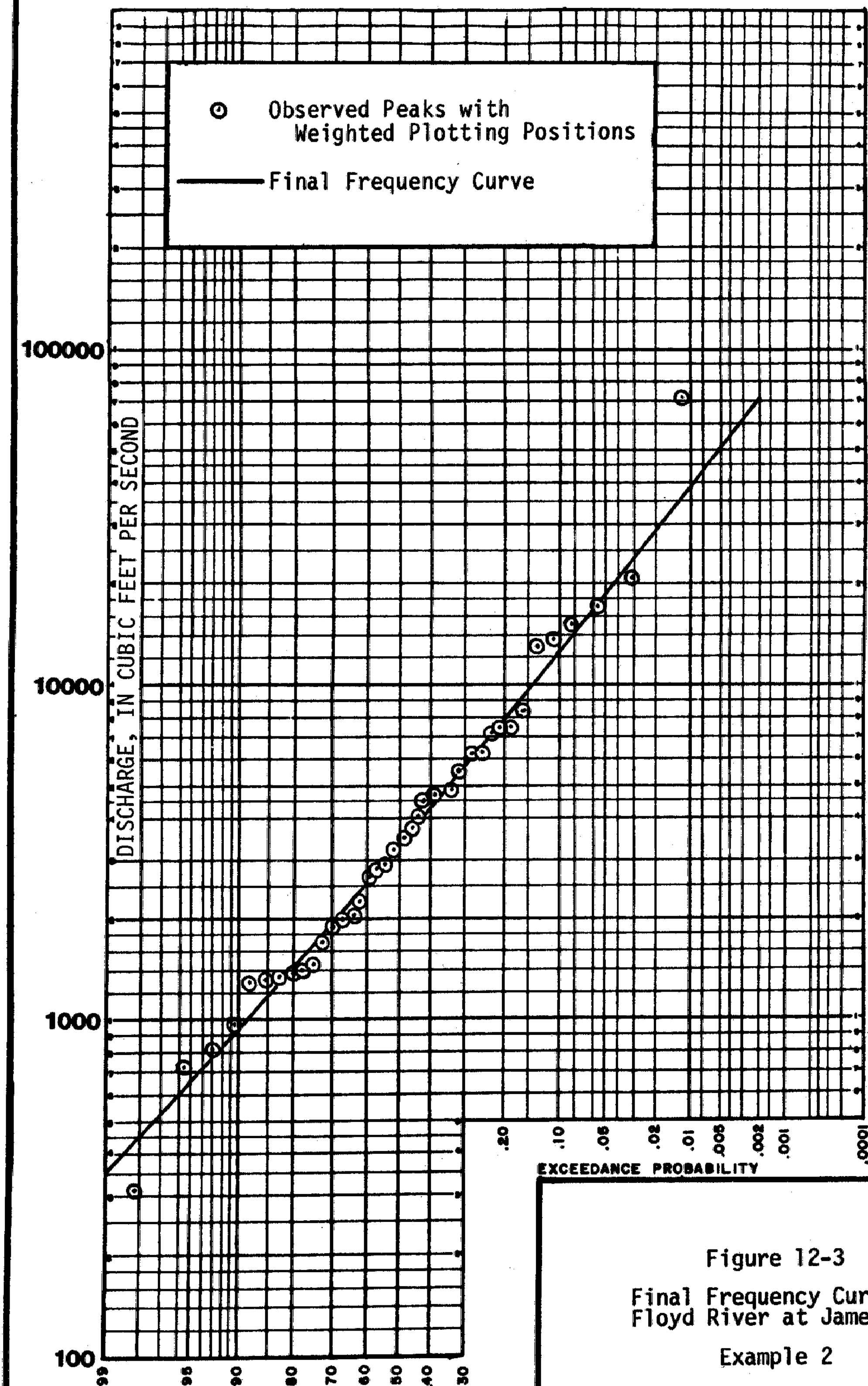


Figure 12-3

Final Frequency Curve for Floyd River at James, Iowa

Example 2

EXAMPLE 3

TESTING AND ADJUSTING FOR A LOW OUTLIER

a. Station Description

Back Creek near Jones Springs, West Virginia

USGS Gaging Station: 01-6140
Lat: $39^{o}30'43''$, long: $78^{o}02'15''$
Drainage Area: 243 sq. mi.
Annual Peaks Available: 1929-31, 1939-1973

b. Computational Procedures

Step 1 - Compute the statistics of the systematic record.

The detailed computations have been omitted; the results of the computations are:

Mean Logarithm	3.7220
Standard Deviation of logs	0.2804
Skew Coefficient of logs	-0.7311
Years	38

At this point the analyst may be interested in seeing the preliminary frequency curve based on the statistics of the systematic record. Figure 12-4 is the preliminary frequency curve based on the computed mean and standard deviation and a weighted skew of -0.2 (based on a generalized skew of 0.5 from Plate I).

Step 2 - Check for outliers.

As the computed skew coefficient is less than -0.4, the test for detecting possible low outliers is made first. From Appendix 4, the K_N for a sample size of 38 is 2.661.

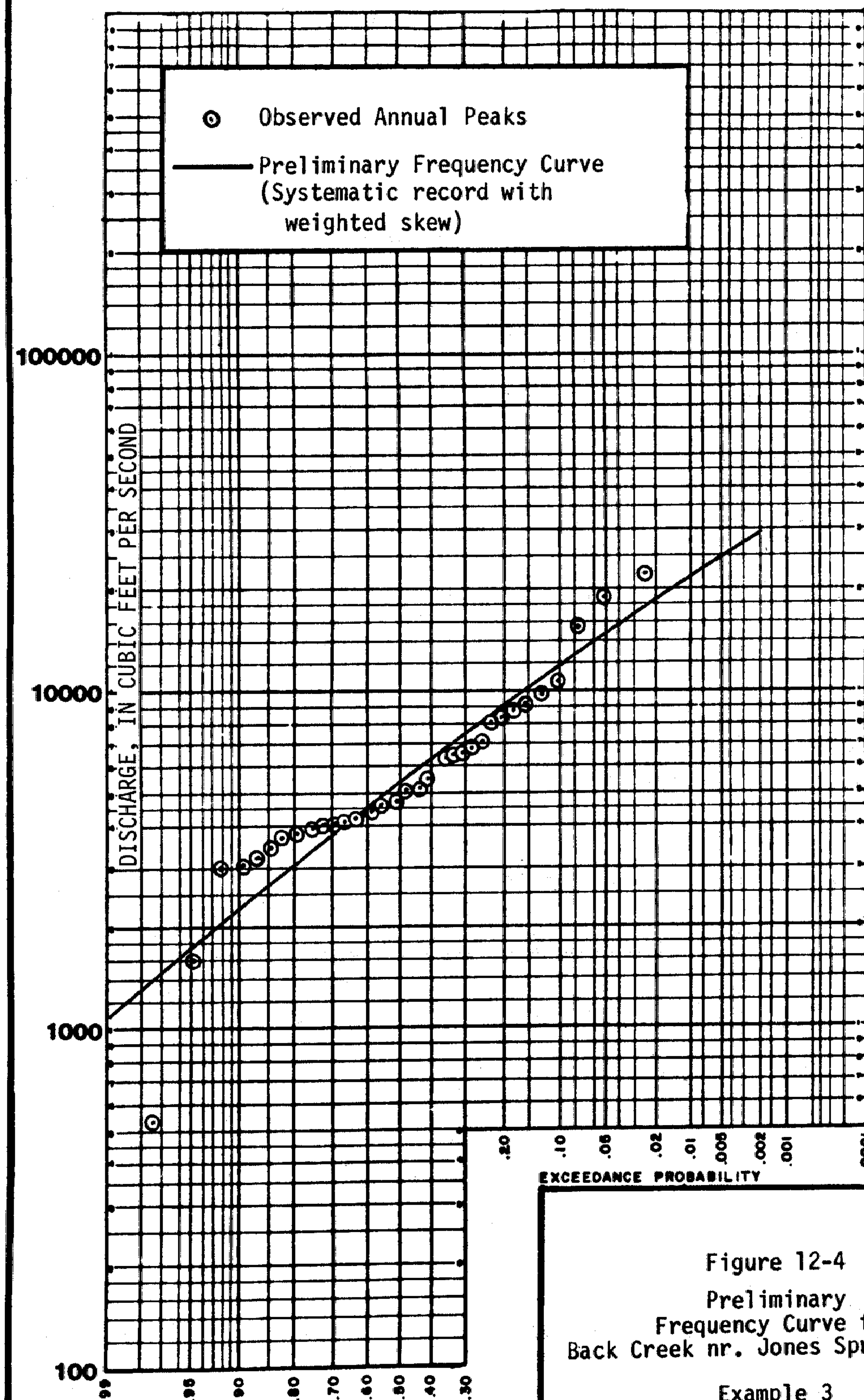


Figure 12-4
Preliminary
Frequency Curve for
Back Creek nr. Jones Springs, W. VA.

Example 3

Example 3 - Testing and Adjusting for a Low Outlier (continued)

The low outlier threshold is computed by Equation 8a:

$$X_L = \overline{X} - K_N S$$

$$= 3.7220 - 2.661\ (.2804) = 2.9759 \qquad (12\text{-}27)$$

$$Q_L = \text{antilog}\ (2.9759) = 946 \text{ cfs}$$

The 1969 event of 536 cfs is below the threshold value of 946 cfs and will be treated as a low outlier.

Step 3 - Delete the low outlier(s) and recompute the statistics.

Mean Logarithm	3.7488
Standard Deviation of logs	0.2296
Skew Coefficient of logs	0.6311
Years	37

Step 4 - Check for high outliers.

The high-outlier threshold is computed to be 22,760 cfs based on the statistics in Step 3 and the sample size of 37 events. No recorded events exceed the threshold value. (See Examples 1 and 2 for the computations to determine the high-outlier threshold.)

Step 5 - Compute and adjust conditional frequency curve.

A conditional frequency curve is computed based on the statistics in Step 3 and then modified by the conditional probability adjustment

(Appendix 5). The skew coefficient has been rounded to 0.6 for ease in computation. The adjustment ratio computed from Equation 5-1a is:

$$\tilde{P} = N/n = 37/38 = 0.9737 \qquad (12\text{-}28)$$

TABLE 12-8

COMPUTATION OF CONDITIONAL FREQUENCY CURVE COORDINATES

P_d	K_{G,P_d} for G = 0.6	log Q	Q cfs	Adjusted Exceedance Probability $(P.P_d)$
.99	-1.88029	3.3171	2080	.9639
.90	-1.20028	3.4732	2970	.876
.50	-0.09945	3.7260	5320	.487
.10	1.32850	4.0538	11300	.097
.05	1.79701	4.1614	14500	.049
.02	2.35931	4.2905	19500	.0195
.01	2.75514	4.3814	24100	.0097
.005	3.13232	4.4680	29400	.0049
.002	3.60872	4.5774	37800	.0019

The conditional frequency curve, along with the adjusted frequency curve, is plotted on Figure 12-5.

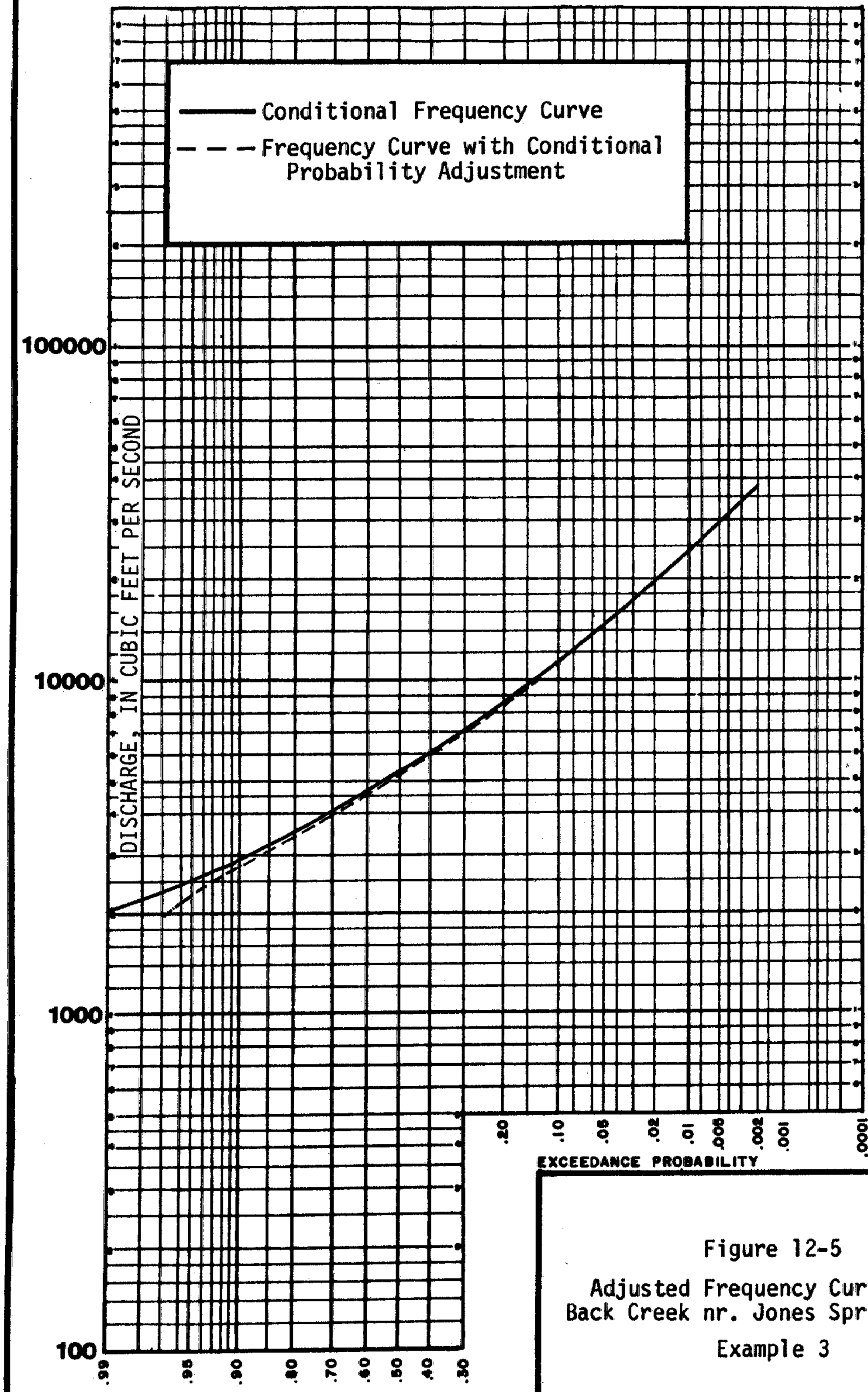


Figure 12-5

Adjusted Frequency Curves for
Back Creek nr. Jones Springs, W. VA.

Example 3

Step 6 - Compute the synthetic statistics.

The statistics of the adjusted frequency curve are unknown. The use of synthetic statistics provides a frequency curve with a log-Pearson Type III shape. First determine the $Q_{.01}$, $Q_{.10}$, and $Q_{.50}$ discharges from the adjusted curve on Figure 12-5.

$$Q_{.01} = 23880 \text{ cfs}$$

$$Q_{.10} = 11210 \text{ cfs}$$

$$Q_{.50} = 5230 \text{ cfs}$$

Next, compute the synthetic skew coefficient by Equation 5-3.

$$G_s = -2.50 + 3.12 \frac{\log(Q_{.01}/Q_{.10})}{\log(Q_{.10}/Q_{.50})}$$

$$= -2.50 + 3.12 \frac{\log(23880/11210)}{\log(11210/5230)} \qquad (12\text{-}29)$$

$$= -2.50 + 3.12 \frac{.32843}{.33110}$$

$$= 0.5948$$

Compute the synthetic standard deviation by Equation 5-4.

$$S_s = \log(Q_{.01}/Q_{.50})/(K_{.01}-K_{.50})$$

$$= \log\ (23880/5230)/\left[2.75514-(-.09945)\right] \qquad (12\text{-}30)$$

$$S_s = .6595/2.8546 = 0.2310$$

Compute the synthetic mean by Equation 5-5.

$$\overline{X}_s = \log\ (Q_{.50}) - K_{.50}(S_s)$$

$$= \log\ (5230) - (-.09945)(.2310) \qquad (12\text{-}31)$$

$$\overline{X}_s = 3.7185 + .0230 = 3.7415$$

Step 7 - Compute the weighted skew coefficient.

The mean-square error of the station skew, from Table 1, is 0.183 based on n = 38 and using G_s for G

$$G_w = \frac{.302(0.5948) + .183(.5)}{.302 + .183} = 0.5590 \qquad (12\text{-}32)$$

$$G_w = 0.6 \text{ (rounded to nearest tenth)}$$

Example 3 - Testing and Adjusting for a Low Outlier (continued)

Step 8 - Compute the final frequency curve.

TABLE 12-9

COMPUTATION OF FREQUENCY CURVE COORDINATES

P	$K_{G_W,P}$ for $G_W = 0.6$	log Q	Q cfs
.99	-1.88029	3.3072	2030
.90	-1.20028	3.4642	2910
.50	-0.09945	3.7185	5230
.10	1.32850	4.0484	11200
.05	1.79701	4.1566	14300
.02	2.35931	4.2865	19300
.01	2.75514	4.3780	23900
.005	3.13232	4.4651	29200
.002	3.60872	4.5751	37600

The final frequency curve is plotted on Figure 12-6

Note: A value of 22,000 cfs was estimated for 1936 on the basis of data from another site. This flow value could be treated as historic data and analyzed by the producers described in Appendix 6. As these computations are for illustrative purposes only, the remaining analysis was not made.

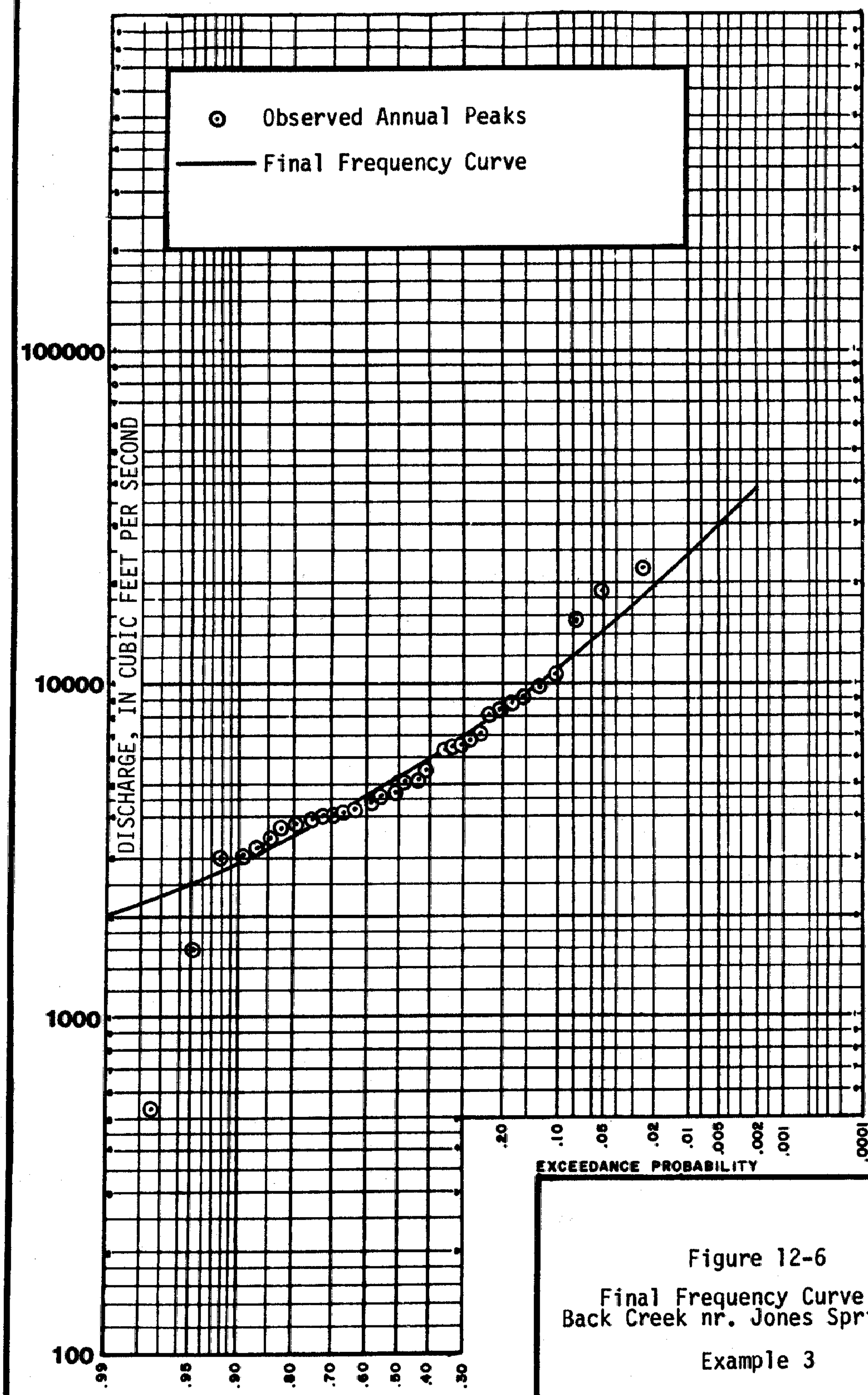


Figure 12-6

Final Frequency Curve for
Back Creek nr. Jones Springs, W. VA.

Example 3

EXAMPLE 4

ADJUSTING FOR ZERO FLOOD YEARS

a. Station Description

Orestimba Creek near Newman, California

USGS Gaging Station: 11-2745
Lat: 37°19'01", long: 121°07'39"
Drainage Area: 134 sq. mi.
Annual Peaks Available: 1932-1973

b. Computational Procedures

Step 1 - Eliminate zero flood years.

There are 6 years with zero flood events, leaving 36 non-zero events.

Step 2 - Compute the statistics of the non-zero events.

Mean Logarithm	3.0786
Standard Deviation of logs	0.6443
Skew Coefficient of logs	-0.8360
Years (Non-Zero Events)	36

Step 3 - Check the conditional frequency curve for outliers.

Because the computed skew coefficient is less than -0.4, the test for detecting possible low outliers is made first. Based on 36 years, the low-outlier threshold is 23.9 cfs. (See Example 3 for low-outlier threshold computational procedure.) The 1955 event of 16 cfs is below the threshold value; therefore, the event will be treated as a low-outlier and the statistics recomputed.

Mean Logarithm	3.1321
Standard Deviation of logs	0.5665
Skew Coefficient of logs	-0.4396
Years (Zero and low outliers deleted)	35

Example 4 - Adjusting for Zero Flood Years (continued)

Step 4 - Check for high outliers

The high outlier threshold is computed to be 41,770 cfs based on the statistics in Step 3 and the sample size of 35 events. No recorded events exceed the threshold value. (See examples 1 and 2 for the computations to determine the high-outlier threshold.)

Step 5 - Compute and adjust the conditional frequency curve.

A conditional frequency curve is computed based on the statistics in step 3 and then adjusted by the conditional probability adjustment (Appendix 5). The skew coefficient has been rounded to -0.4 for ease in computation. The adjustment ratio is 35/42 = 0.83333.

TABLE 12-10

COMPUTATION OF CONDITIONAL FREQUENCY CURVE COORDINATES

P_d	$K_{G,P}$ for G = -0.4	log Q	Q cfs	Adjusted Exceedance Probability ($\tilde{P}.P_d$)
.99	-2.61539	1.6505	44.7	.825
.90	-1.31671	2.3862	243	.750
.50	0.06651	3.1698	1480	.417
.10	1.23114	3.8295	6750	.083
.05	1.52357	3.9952	98900	.042
.02	1.83361	4.1708	14800	.017
.01	2.02933	4.2817	19100	.0083
.005	2.20092	4.3789	23900	.0042
.002	2.39942	4.4914	31000	.0017

Both frequency curves are plotted on Figure 12-7.

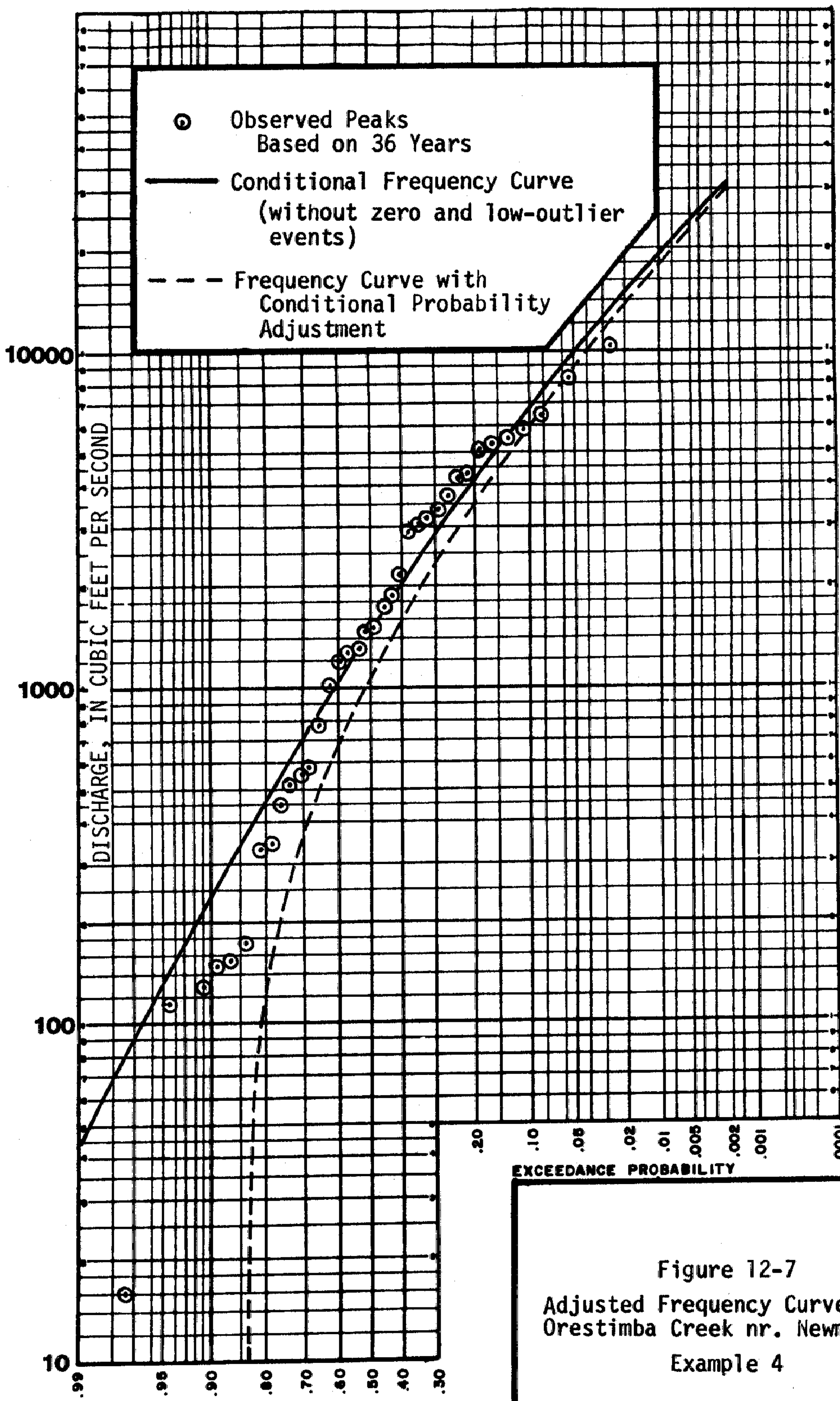


Figure 12-7
Adjusted Frequency Curves for
Orestimba Creek nr. Newman, CA
Example 4

Example 4 - Adjusting for Zero Flood Years (continued)

Step 6 - Compute the synthetic statistics.

First determine the $Q_{.01}$, $Q_{.10}$, and $Q_{.50}$ discharges from the adjusted curve on Figure 12-7.

$$Q_{.01} = 17940 \text{ cfs}$$

$$Q_{.10} = 6000 \text{ cfs}$$

$$Q_{.50} = 1060 \text{ cfs}$$

Compute the synthetic skew coefficient by Equation 5-3.

$$G_s = -2.50 + 3.12 \frac{\log(17940/6000)}{\log(6000/1060)} = -0.5287 \qquad (12\text{-}33)$$

$$G_s = -0.5 \text{ (rounded to nearest tenth)}$$

Compute the synthetic standard deviation by Equation 5-4.

$$S_s = \log(17940/1060)/(1.95472 - .08302) \qquad (12\text{-}34)$$

$$S_s = 0.6564$$

Compute the synthetic mean by Equation 5-5.

$$\overline{X}_s = \log(1060) - (.08302)(.6564) \qquad (12\text{-}35)$$

$$\overline{X}_s = 2.9708$$

Step 7 - Compute the weighted skew coefficient by Equation 5.

A generalized skew of -0.3 is determined from Plate I. From Table I, the mean-square error of the station skew is 0.163.

$$G_w = \frac{.302(-.529) + .163(-.3)}{.302 + .163} = -0.4487 \qquad (12\text{-}36)$$

$$G_w = -0.4 \text{ (rounded to nearest tenth)}$$

Example 4 - Adjusting for Zero Flood Years (continued)

Step 8 - Compute the final frequency curve.

TABLE 12-11

COMPUTATION OF FREQUENCY CURVE ORDINATES

P	$K_{G_W,P}$ for G_W = -0.4	log Q	Q cfs
.99	-2.61539	1.2541	17.9
.90	-1.31671	2.1065	128
.50	0.06651	3.0145	1030
.10	1.23114	3.7789	6010
.05	1.52357	3.9709	9350
.02	1.83361	4.1744	14900
.01	2.02933	4.3029	20100
.005	2.20092	4.4155	26000
.002	2.39942	4.5458	35100

This frequency curve is plotted on Figure 12-8. The adjusted frequency derived in Step 4 is also shown on Figure 12-8. As the generalized skew may have been determined from stations with much different characteristics from the zero flood record station, judgment is required to determine the most reasonable frequency curve.

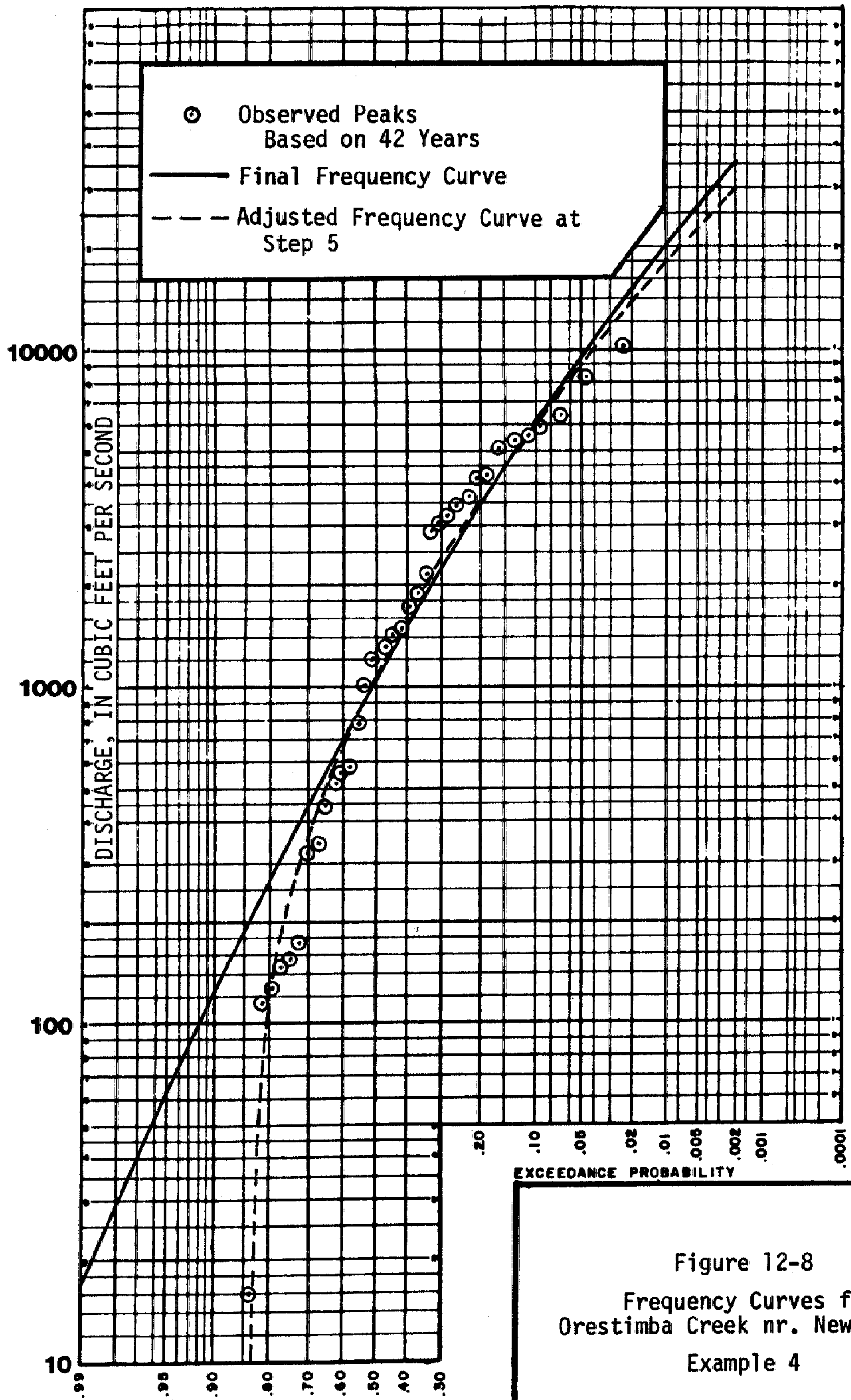


Figure 12-8

Frequency Curves for Orestimba Creek nr. Newman, CA

Example 4

Appendix 13

COMPUTER PROGRAM

+ Programs have been developed that compute a log-Pearson Type III distribution from systematically recorded annual maximum streamflows at a single station -- and other large known events. Special routines are included for managing zero flows and very small flows (outliers) that would distort the curve in the range of higher flows. An option is included to adjust the computed curve to represent expected probability. Copies of agency programs that incorporate procedures recommended by this Guide may be obtained from either of the following:

Chief Hydrologist
U.S. Geological Survey, WRD
National Center, Mail Stop 437
Reston, VA 22092

Phone: (703) 860-6879

Hydrologic Engineering Center
U.S. Army Corps of Engineers
609 2nd Street, Suite I
Davis, CA 95616

Phone: (916) 756-1104

There is no specific recommendation to utilize these particular computer programs. Other federal and state agencies as well as private organizations may have developed individual programs to suit their specific needs. +

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Appendix 14

"FLOOD FLOW FREQUENCY TECHNIQUES"

REPORT SUMMARY

*

Following is a summary of "Flood Flow Frequency Techniques," a report by Leo R. Beard, Technical Director, Center for Research in Water Resources, The University of Texas at Austin, for the Office of Water Resources Research and the Water Resources Council. Much of the text and a majority of the exhibits are taken directly from the report.

The study was made at the Center for Research in Water Resources of The University of Texas at Austin at the request of and under the general guidance of the Work Group on Flood Flow Frequency, Hydrology Committee, of the Water Resources Council through the auspices of the Office of Water Resources Research. The purpose was to provide a basis for development by the Work Group of a guide for flood frequency analysis at locations where gage records are available which would incorporate the best technical methods currently known and would yield greater reliability and consistency than has heretofore been available in flood flow frequency determinations.

The study included: (a) a review of the literature and current practice to select candidate methods and procedures for testing, (b) selection of long-record station data of natural streamflows in the United States and development of data management and analysis computer programs for testing alternate procedures, (c) testing eight basic statistical methods for frequency analysis including alternate distributions and fitting techniques, (d) testing of alternate criteria for managing outliers, (e) testing of procedures for treating stations with zero flow years, (f) testing relationships between annual maximum and partial-duration series, (g) testing of expected probability adjustment, (h) testing to determine if flood data exhibit consistent long-term trends, and (i) recommendations with regard to each procedure tested and development of background material for the guides being developed by the Work Group.

Data

In all, 300 stations were used in the testing. Flows were essentially unregulated. Record length exceeded 30 years with most stations having records longer than 40 years. The stations were selected to give the best feasible coverage of drainage area size and geographic location and to include a substantial number of stations with no flow for an entire year. Table 14-1 lists the number of stations by size and geographic zone.

Split Record Testing

A primary concern of the study was selection of a mathematical function and fitting technique that best estimates flood flow frequencies from annual peak flow data. Goodness of fit of a function to the data used in the fitting process is not necessarily a valid criterion for selecting a method that best estimates flood frequencies. Consequently, split record testing was used to simulate conditions of actual application by reserving a portion of a record from the fitting computation and using it as "future" events that would occur in practice. Goodness of fit can nevertheless be used, particularly to eliminate methods whose fit is very poor.

Each record of annual maximum flows was divided into two halves, using odd sequence numbers for one half and even for the other in order to eliminate the effect of any general trend that might possibly exist. This splitting procedure should adequately simulate practical situations as annual events were tested and found independent of each other. Frequency estimates were made from each half of a record and tested against what actually happened in the other half.

Development of verification criteria is complicated, because what actually happens in the reserved record half also is subject to sampling irregularities. Consequently, reserved data cannot be used as a simple, accurate target and verification criteria must be probabilistic. The test procedure, however, simulates conditions faced by the planner, designer, or operator of water resource projects, who knows neither that past events are representative nor what future events will be.

The ultimate objective of any statistical estimation process is not to estimate the most likely theoretical distribution that generated the observed data, but rather to best forecast future events for which a decision is formulated. Use of theoretical distribution functions and their attendant reliability criteria is ordinarily an intermediate step to forecasting future events. Accordingly, the split record technique of testing used in this study should be more rigorous and direct than alternative theoretical goodness-of-fit tests.

Frequency Computation Methods

Basic methods and fitting techniques tested in this study were selected by the author and the WRC Work Group on Flood Flow Frequency after careful review of the literature and experience in the various agencies represented; those that were tested are listed below. Numbering corresponds to the identification number of the methods in the computer programs and in the attached tables.

1. Log-Pearson Type III (LP3). The technique used for this is that described in (35). The mean, standard deviation, and skew coefficients for each data set are computed in accordance with the following equations:

$$\overline{X} = \frac{\Sigma X}{N} \qquad (14\text{-}1)$$

$$S^2 = \frac{\Sigma X^2 - (\Sigma X)^2/N}{N-1} \qquad (14\text{-}2)$$

$$g = \frac{N^2 \Sigma X^3 - 3N \Sigma X \Sigma X^2 + 2(\Sigma X)^3}{N(N-1)\ (N-2) S^3} \qquad (14\text{-}3)$$

where

X = logarithm of peak flow

N = number of items in the data set

$\overline{X}$ = mean logarithm

S = standard deviation of logarithms

g = skew coefficient of logarithms

Flow logarithms are related to these statistics by use of the following equation:

$$X = \overline{X} + kS \qquad (14\text{-}4)$$

Exceedance probabilities for specified values of k and values of k for specified exceedance probabilities are calculated by use of the normal distribution routines available in computer libraries and the approximate transform to Pearson deviates given in reference (31).

2. Log Normal (LN). This method uses a 2-parameter function identical to the log-Pearson III function except that the skew coefficient is not computed (a value of zero applies), and values of k are related to exceedance probabilities by use of the normal distribution transform available in computer libraries.

3. Gumbel (G). This is the Fisher-Tippett extreme-value function, which relates magnitude linearly with the log of the log of the reciprocal of exceedance probability (natural logarithms). Maximum likelihood estimates of the mode and slope (location and scale parameters) are made by iteration using procedures described by Harter and Moore in reference (36). The initial estimates of the location and scale statistics are obtained as follows:

$$M = \overline{X} - 0.45005\ S \qquad (14\text{-}5)$$

$$B = .7797\ S \qquad (14\text{-}6)$$

Magnitudes are related to these statistics as follows:

$$X = M + B(-\ln(-\ln P)) \qquad (14\text{-}7)$$

where

M = mode (location statistic)
B = slope (scale statistic)
X = magnitude
P = exceedance probability
S = standard deviation of flows

Some of the computer routines used in this method were furnished by the Central Technical Unit of the Soil Conservation Service.

4. Log Gumbel (LG). This technique is identical to the Gumbel technique except that logarithms (base 10) of the flows are used.

5. Two-parameter Gamma (G2). This is identical to the 3-parameter Gamma method described below, except that the location parameter is set to zero. The shape parameter is determined directly by solution of Nörlund's (37) expansion of the maximum likelihood equation which gives the following as an approximate estimate of α:

$$\alpha = 1 + \frac{\sqrt{1 + \frac{4}{3}\left(\ln\overline{Q} - \frac{1}{N}\Sigma\ln Q\right)}}{4\left(\ln\overline{Q} - \frac{1}{N}\Sigma\ln Q\right)} - \Delta\alpha \qquad (14\text{-}8)$$

where

$\overline{Q}$ = average annual peak flow

N = number of items in the data set

Q = peak flow

$\Delta\alpha$ = correction factor

β is estimated as follows:

$$\beta = \frac{1}{\alpha} \cdot \frac{1}{N}\Sigma Q \qquad (14\text{-}9)$$

6. Three-parameter Gamma (G3). Computation of maximum likelihood statistics for the 3-parameter Gamma distribution is accomplished using procedures described in reference (38). If the minimum flow is zero, or if the calculated lower bound is less than zero, the statistics are identical to those for the 2-parameter Gamma distribution. Otherwise, the lower bound, γ, is initialized at a value slightly smaller than the lowest value of record, and the maximum likelihood value of the lower bound is derived by iteration using criteria in reference (38). Then the parateters α and β are solved for directly using the equations above replacing Q with Q-γ. Probabilities corresponding to specified magnitudes are computed directly by use of a library gamma routine. Magnitudes corresponding to specified

probabilities are computed by iteration using the inverse solution.

7. Regional Log-Pearson Type III (LPR). This method is identical to the log-Pearson Type III method, except that the skew coefficient is taken from Figure 14-1 instead of using the computed skew coefficient. Regionalized skew coefficients were furnished by the U.S. Geological Survey.

8. Best Linear Invariant Gumbel (BLI). This method is the same as for the Gumbel method, except that best linear invariant estimates (BLIE) are used for the function statistics instead of the maximum likelihood estimates (MLE). An automatic censoring routine is used for this method only, so there are no altenative outlier techniques tested for this method. Statistics are computed as follows:

$$M = \Sigma(X(I) \cdot U(N,J,I)) \tag{14-10}$$

$$B = \Sigma(X(I) \cdot V(N,J,I)) \tag{14-11}$$

where

U = coefficient UMANN described in reference (39)
V = coefficient BMANN described in reference (39)
J = number of outliers deleted plus 1
I = order number of flows arranged in ascending-magnitude order
N = sample size as censored.

Since weighting coefficients U and V were made available in this study only for sample sizes ranging from 10 to 25, 5-year samples are not treated by this method, and records (or half records) of more than 25 years are divided into chronological groups and weighted average coefficients used in lieu of coefficients that might otherwise be obtained if more complete sets of weighting coefficients were available. Up to two outliers are censored at the upper end of the flow array. Each one is removed if sequential tests show that a value that extreme would occur by chance less than 1 time 10 on the basis of the BLIE statistics. Details of this censoring technique are contained in reference (40). Weighting coefficients and most of the routines used in this method were furnished by the Central Technical Unit of the Soil Conservation Service.

Outliers

Outliers were defined for purpose of this study as extreme values whose ratio to the next most extreme value in the same (positive or negative) direction is more extreme than the ratio of the next most extreme value to the eighth most extreme value.

The techniques tested for handling outliers consisted of

a. keeping the value as is,

b. reducing the value to the product of the second largest event and the ratio of the second largest to eighth largest event,

c. reducing the value to the product of the second largest event and the square root of that ratio, and

d. discarding the value.

In the cases of outliers at the low end, the words largest in (b) and (c) should be changed to smallest.

Zero Flow

Two techniques were tested for handling stations with some complete years of no flow as follows:

(a) Adding 1 percent of the mean magnitude to all values for computation purposes and subtracting that amount from subsequent estimates, and

(b) removing all zeros and multiplying estimated exceedance frequencies of the remaining by the ratio of the number of non-zero values to the total number of values. This is the procedure of combining probabilities described in reference (27).

Partial-Duration Series

A secondary concern of the study was the relationship between annual maximum flow frequencies and partial-duration flow frequencies.

Because a partial-duration series consists of all events above a specified magnitude, it is necessary to define separate events. The definition normally depends on the application of the frequency study as well as the hydrologic characteristics of the stream. For this study separate events were arbitrarily defined as events separated by at least as many days as five plus the natural logarithm of the square miles of drainage area, with the requirement that intermediate flows must drop below 75 percent of the lower of the two separate maximum daily flows. This is considered representative of separation criteria appropriate for many applications.

Maximum daily flows were used for this part of the study, because there were insufficient readily available data on instantaneous peak flows for events smaller than the annual maximum. There is no reason to believe that the frequency relationship would be different for peak flows than for daily flows.

The relationship between the maximum annual and partial-duration series was expressed as a ratio of partial-duration to annual event frequencies at selected annual event frequencies. In order to develop partial-duration relationships independent of any assumptions as to frequency functions, magnitudes corresponding to annual-maximum event exceedance probabilities of 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, and 0.7 are established for complete records at each station by linear interpolation between expected probability plotting positions (M/(n+1)) for the annual maximum events. Corresponding frequencies of partial-duration flows are established simply by counting the total number of independent maximum daily flows at each station above each magnitude and dividing by the total number of years at that station. Ratios of partial-duration to annual event frequencies were averaged for all stations in each USGS zone and compared with ratios derived for certain theoretical conditions by Langbein (9).

Expected Probability Estimation

The expected probability is defined as the average of the true probabilities of all magnitude estimates for any specified flood frequency that might be made from successive samples of a specified size. For any specified flow magnitude, it is considered to be the most appropriate estimate of probability or frequency of future flows for water resources planning and management use.

It is also a probability estimate that is theoretically easy to verify, because the observed frequencies in reserved data at a large number of stations should approach the computed probability or frequency estimates as the number of stations increases. Accordingly, it was considered that expected probability estimates should be used in the split record tests.

A method of computing expected probabilities has been developed for samples drawn from a Gaussian normal distribution as described in (21).

Similar techniques are not available for the other threoretical distribution functions. Consequently, an empirical transform is derived for each distribution. To do this a calibration constant was determined which, when multiplied by the theoretical normal transform adjustment, removed the observed average bias in estimating probabilities for the 300 stations used in this study. This empirical transform was used in making the accuracy tests that are the main basis for judging the relative adequacy of the various methods tests.

Trends and Cycles

There is some question as to whether long-term trends and cycles (longer than 1 year) exist in nature such that knowledge of their nature can be used to improve forecasts of flood flow frequencies for specific times in the future. As a part of this research project, lag 1 autocorrelation coefficients of annual peak flows for all stations were computed. If trends or cycles exist in any substantial part of the data, there should be a net positive average autocorrelation for all stations. A statistically significant positive average autocorrelation was not found.

Accuracy and Consistency Tests

Criteria used in judging the adequacy of each method for fitting a theoretical distribution were as follows:

Accuracy tests consisted of the following comparisons between computed frequencies in one-half the record with frequencies of events that occurred in the reserved data.

a. Standard deviation of observed frequencies (by count) in reserved data for magnitude estimates corresponding to exceedance probabilities of 0.001, 0.01, 0.1, and 0.5 computed from the part of the record used. This is the standard error of a frequency estimate at individual stations that would occur if a correction is made for the average observed bias in each group of stations for each selected frequency and method.

b. Root-mean-square difference between expected probability plotting position (M/(n+1)) of the largest, upper decile and median event in a half record and the computed expected probability exceedance frequency of that respective event in the other half. This is the standard error of a frequency estimate at individual stations without any bias adjustment for each method and for the frequency of each selected event.

c. Root-mean-square difference between 1.0 and the ratio of the computed probability of flow in the opposite half of a record to the plotting position of the largest, upper decile and median event (in turn) in a half record. This criterion is similar to that of the preceding paragraph except that methods that are biased toward predicting small frequencies are not favored.

Consistency tests involved the following comparisons between computed frequencies in each half of the record with the total record.

a. Root-mean-square difference between computed probabilities from the two record halves for full record extreme, largest, upper decile and median events, in turn. This is an indicator of the relative uniformity of estimates that would be made with various random samples for the same location.

b. Root-mean-square value of 1.0 minus the ratio of the smaller to the larger computed probabilities from the two record halves for full record extreme, largest, upper decile and median events, in turn. This is essentially the same as the preceding criterion, except that methods that are biased toward predicting small frequencies are not favored.

The extreme event used in the consistency tests is an arbitrary value equal to the largest multiplied by the square root of the ratio of the largest to the median event for the full record.

It should be recognized that sampling errors in the reserved data are as large or larger for the same sample size as are sampling errors of computed values. Similarly, sampling errors are comparable for estimates based on opposite record halves used for consistency tests. Consequently, a great number of tests is necessary in order to reduce the uncertainty due to sampling errors in the reserved data. Further, a method that is biased toward estimating frequencies too low may have a small standard error of estimating frequencies in comparison with a method that is biased toward high frequencies, if the bias is not removed. The latter may have smaller percentage errors. Accordingly, consideration of the average frequency estimate for each of the eight methods must be a component of the analyses.

As a further means of evaluating alternate procedures the complete record results, computed curve without any expected probability adjustment, and the plotted data point were printed out.

Evaluation of Distributions

Table 14-2 shows for each method and each USGS zone the number of stations where an observed discharge exeeeded the computed 1,000-year discharge. With 14,200 station-years of record, it might be expected that about 14 observed events would exceed true 1,000-year magnitudes. This comparison indicates that the log-Pearson Type III (method 1), log normal (method 2), and log-Pearson Type III with generalized skew (method 7), are the most accurate.

Table 14-3 shows average observed frequencies (by count) in the reserved portions of half records for computed probabilities of 0.001, 0.01, 0.1, and 0.5 and the standard deviations (accuracy test a) of the observed frequencies from their averages for each computed frequency. It is difficult to draw conclusions from these data. Figure 14-2 shows a plotting of the results for the 0.01 probability estimates which aids in comparison. This comparison indicates that the log normal and log-Pearson Type III methods with generalized skew have observed frequencies closest to those computed and the smallest standard deviations except for method 4.

Table 14-4 shows the average results for all stations of accuracy tests b and c. Results are not definitive, but again the log normal (method 2) and log-Pearson Type III with generalized skew (method 7) show results as favorable as any other method as illustrated for test b in Figure 14-3.

Table 14-5 shows the results of the consistency tests. Figure 14-4 displays the results graphically for test a. The consistency test results are not substantially different from or more definitive than the accuracy results. From Figure 14-4 it appears that the log-Pearson Type III method with generalized skew yields considerably more consistent results than the log normal.

Results of Outlier Testing

Table 14-6 shows results for all stations of the accuracy and consistency tests for the four different outlier techniques. Results of these tests show that for the favorable methods [log normal (method 2) and log-Pearson Type III with generalized skew (method 7)], outlier techniques a and b are most favorable. Unfortunately, no discrimination was made in the verification tests between treatment of outliers at the upper and lower ends of the frequency arrays. Outliers at the lower end can greatly increase computed frequencies at the upper end. Average computed frequencies for all half records having outliers at the upper or lower end are generally high for the first three outlier techniques and low for the fourth.

It is considered that this is caused primarily by outliers at the lower end. Values observed are as follows:

Average plotting position of maximum flow	0.042
Average computed probability, method a	0.059
Average computed probability, method b	0.050
Average computed probability, method c	0.045
Average computed probability, method d	0.038

Until more discriminatory outlier studies are made, method a appears to be the most logical and justifiable to use.

Results of Zero Flow Testings

Table 14-7 shows the average for all stations of the results of accuracy and consistency tests for the two different zero flow techniques.

These test comparisons indicate that for the favorable methods [log normal (method 2) and log-Pearson Type III with generalized skew (method 7)], technique b is slightly better than a.

Results of Partial-Duration Studies

Results of partial-duration studies are shown in Table 14-8. It can be seen that there is some variation in values obtained for different zones and that the average of all zones is somewhat greater than the theoretical values developed by Langbein. The theoretical values were based on the assumption that a large number of independent (random) events occur each year. If the number of events per year is small, the average values in Table 14-8 would be expected to be smaller than the theoretical values. If the events are not independent such that large events tend to cluster in some years and small events tend to cluster in other years, the average values in Table 14-8 would be expected to be larger than the theoretical values.

It was concluded that values computed for any given region (not necessarily zones as used in this study) should be used for stations in that region after smoothing the values such that they have a constant relation to the Langbein theoretical function.

Expected Probability Adjustment Results

The ratios by which the normal expected probability theoretical adjustment must be multiplied in order to compute average probabilities equal to those observed for each zone are shown in Tables 14-9, 14-10, and 14-11. It will be noted that these vary considerably from zone to zone and for different exceedance intervals. Much of this variation, however, is believed due to vagaries of sampling. Average ratios for the 100-year flood shown on the last line in Table 14-10 were adopted for each distribution for the purpose of comparing accuracy and the various methods. These are as follows:

1.	Log-Pearson Type III	2.1
2.	Log Normal	0.9
3.	Gumbel, MLE	3.4
4.	Log Gumbel	-1.2
5.	2-parameter gamma	3.4

6. 3-parameter gamma 2.3
7. Regional log-Pearson Type III 1.1
8. Gumbel, BLIE 5.7

Results of this portion of the study indicate that only the log normal (method 2) and log-Pearson Type III with regional skew (method 7) are free of substantial bias because zero bias should correspond approximately to a coefficient of 1.0 as would be the case if the distribution characteristics do not greatly influence the adjustment factor. The following tabulation for log-Pearson Type III method with regional skew indicates that the theoretical expected probability adjustment for the normal distribution applies approximately for this method. Coefficients shown range around the theoretical value of 1.0 and, with only one exception, do not greatly depart from it in terms of standard-error multiples. It is particularly significant that the most reliable data (the 100-year values) indicate an adjustment factor near 1.0.

Expected Probability Adjustment Ratios for All Zones

Sample	10-Yr		100-Yr		1000-Yr	
Size	Avg.	Std. Err.	Avg.	Std. Err.	Avg.	Std. Err.
5	0.81	0.17	0.94	0.12	1.01	0.13
10	0.60	0.22	1.12	0.20	1.45	0.27
23	0.17	0.27	1.14	0.23	1.68	0.28

Results of Test for Trends and Cycles

Results of lag I autocorrelation studies to test for trends are shown in Table 14-12. It is apparent that there is a tendency toward positive autocorrelation, indicating a tendency for flood years to cluster more than would occur in a completely random process. The t values shown are multiples of the standard error of the lag I correlation coefficient, and it is obvious that extreme correlation coefficients observed are not seriously different from variations that would occur by chance. It is considered that annual peak flows approximate a random process in streams used in this study.

Conclusions

Although split record results were not as definitive as anticipated, there are sufficient clearcut results to support definite recommendations. Conclusions that can be drawn are as follows:

a. Only method 2 (log normal) and method 7 (log-Pearson Type III with regional skew) are not greatly biased in estimating future frequencies.

b. Method 7 gives somewhat more consistent results than method 2.

c. For methods 2 and 7, outlier technique "a" (retaining the outlier as recorded) is more accurate in terms of ratio of computed to observed frequencies than methods that give less weight to outliers.

d. For methods 2 and 7, zero flow technique "b" (discarding zero flows and adjusting computed frequencies) is slightly superior to zero flow technique "a."

e. Streamflows as represented by the 300 stations selected for this study are not substantially autocorrelated; thus, records need not be continuous for use in frequency analysis.

f. Partial-duration frequencies are related to annual event frequencies differently in different regions; thus, empirical regional relationships should be used rather than a single theoretical relationship.

Of particular significance is the conclusion that frequencies computed from theoretical functions in the classical manner must be adjusted to reflect more frequent extreme events if frequencies computed in a great number of cases are to average the same as observed frequencies. For the recommended method, adjustment equal to the theoretical adjustment for estimates made from samples drawn from a normal population is approximately correct.

Of interest from a research standpoint is the finding that split record techniques require more than 300 records of about 50 events each to be definitive. This study showed that random variations in the reserved data obscure the results to greater degree than would be the case if curve-fitting functions could reduce uncertainty to a greater degree than has been possible.

In essence, then, regardless of the methodology employed, substantial uncertainty in frequency estimates from station data will exist, but the log-Pearson type III method with regional skew coefficients will produce unbiased estimates when the adjustment to expected probability is employed, and will reduce uncertainty as much as or more than other methods tested.

Recommendations for Future Study

It is considered that this study is an initial phase of a more comprehensive study that should include

a. Differentiation in the treatment of outliers at the upper and lower ends of a frequency curve;

b. Treatment of sequences composed of different types of events such as flood flows resulting from rainfall and those from snowmelt, or hurricane and nonhurricane floods;

c. Physical explanation for great differences in frequency characteristics among streams in a given region;

d. Development of systematic procedures for regional coordination of flood flow frequency estimates and applications to locations with recorded data as well as to locations without recorded data;

e. Development of procedures for deriving frequency curves for modified basin conditions, such as by urbanization;

f. Development of a step-by-step procedure for deriving frequency curves for locations with various amounts and types of data such that progressively reliable results can be obtained on a consistent basis as the amount of effort expended is increased; and

g. Preparation of a text on flood flow frequency determinations for use in training and practical application.

FIGURE 14-1

GENERALIZED SKEW COEFFICIENTS OF ANNUAL MAXIMUM STREAMFLOW LOGARITHMS

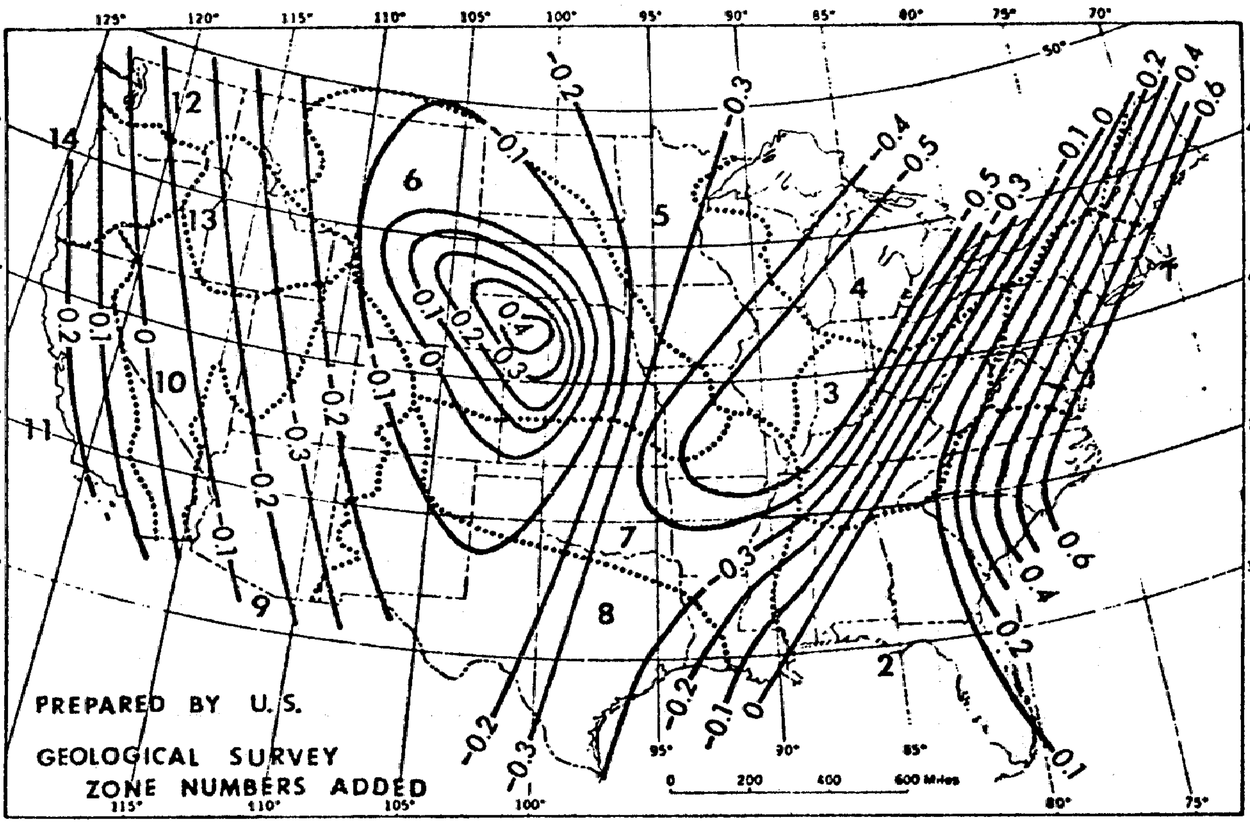


FIGURE 14-2

ACCURACY COMPARISON FOR 0.01 PROBABILITY ESTIMATE (TABLE 14-3)

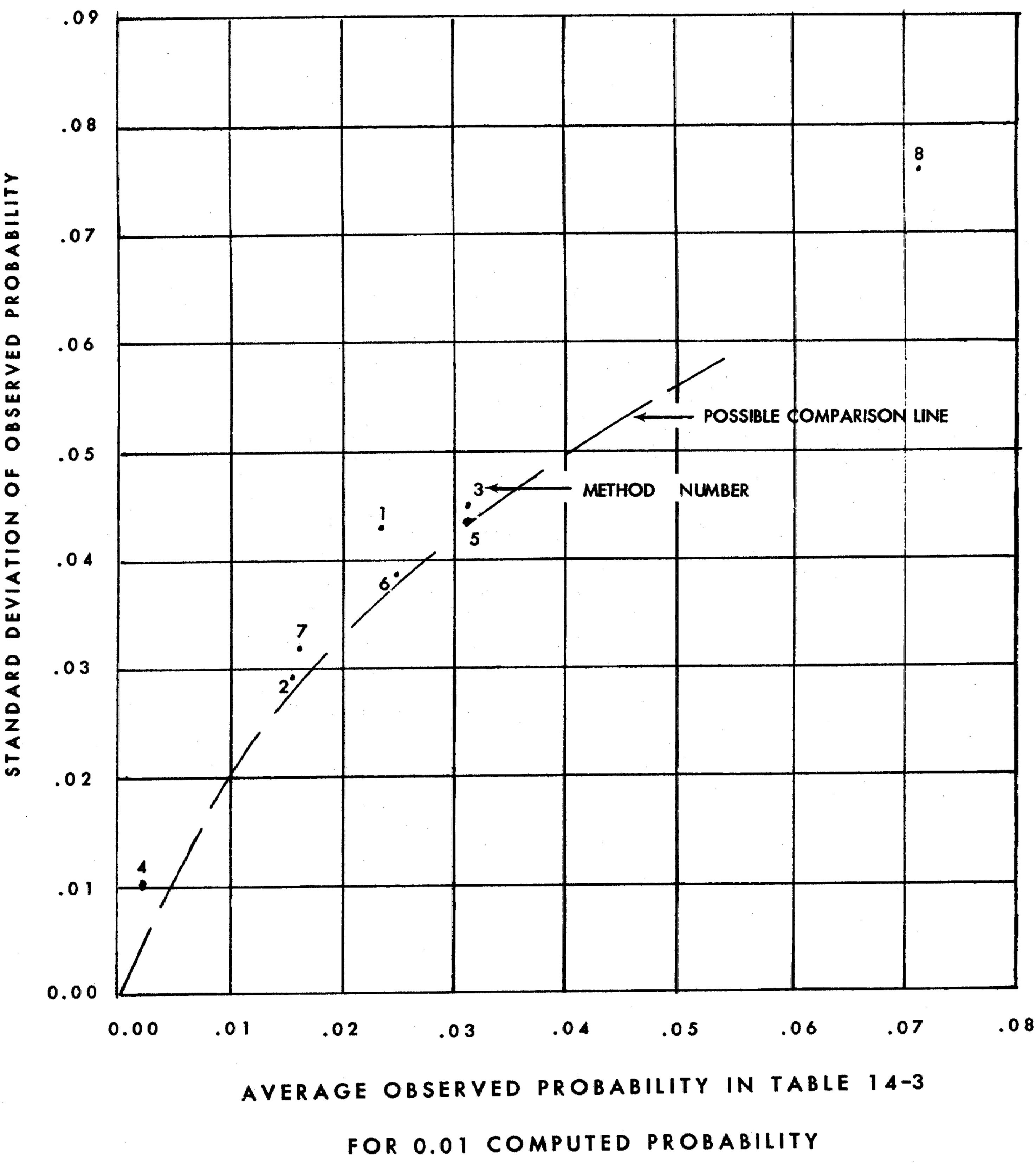
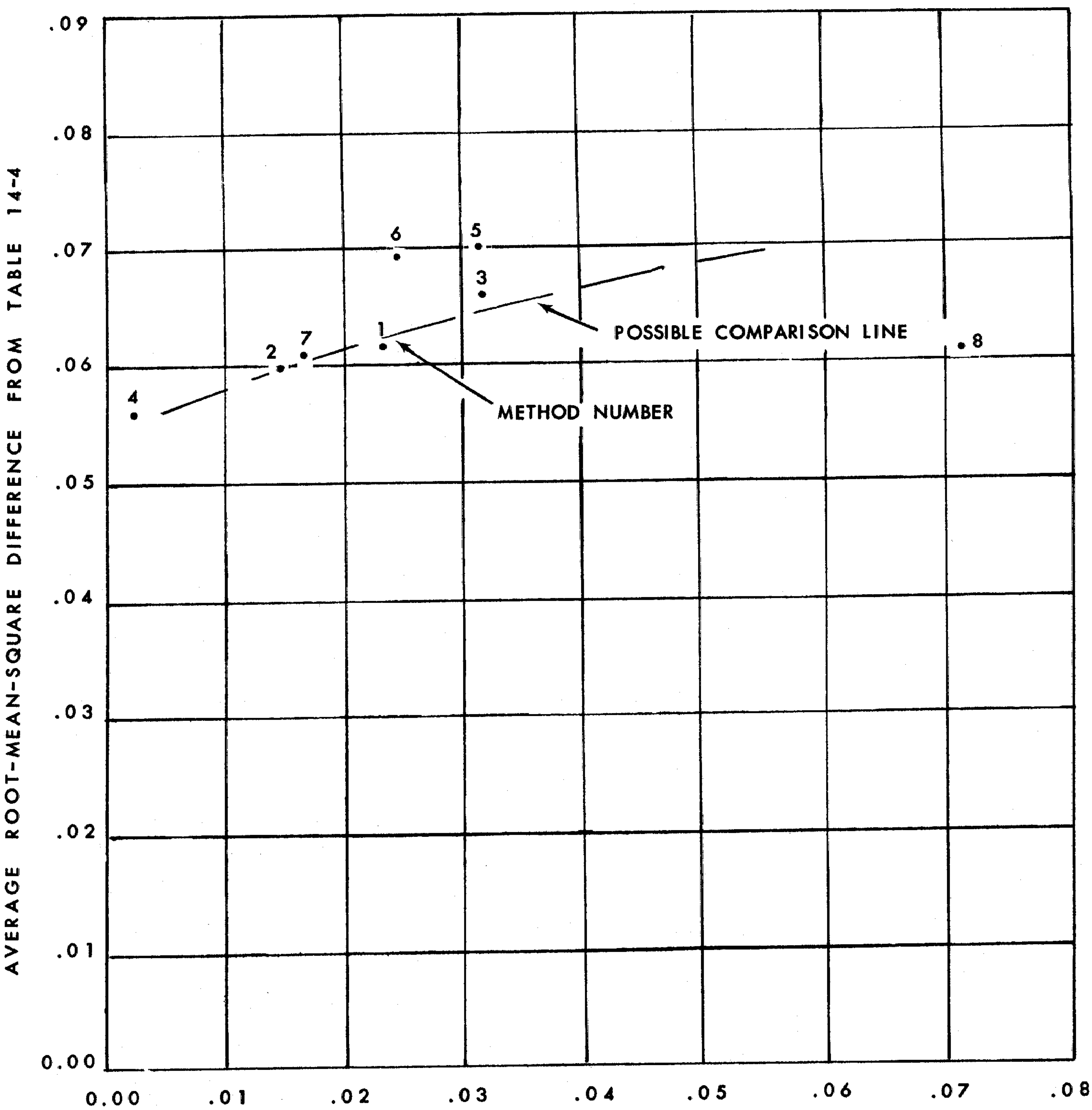


FIGURE 14-3

ACCURACY COMPARISON FOR MAXIMUM OBSERVED FLOW

(TABLE 14-4, TEST B)



AVERAGE OBSERVED PROBABILITY IN TABLE 14-3

FOR 0.01 COMPUTED PROBABILITY

FIGURE 14-4

CONSISTENCY COMPARISON FOR MAXIMUM OBSERVED FLOW

(TABLE 14-5, TEST A)

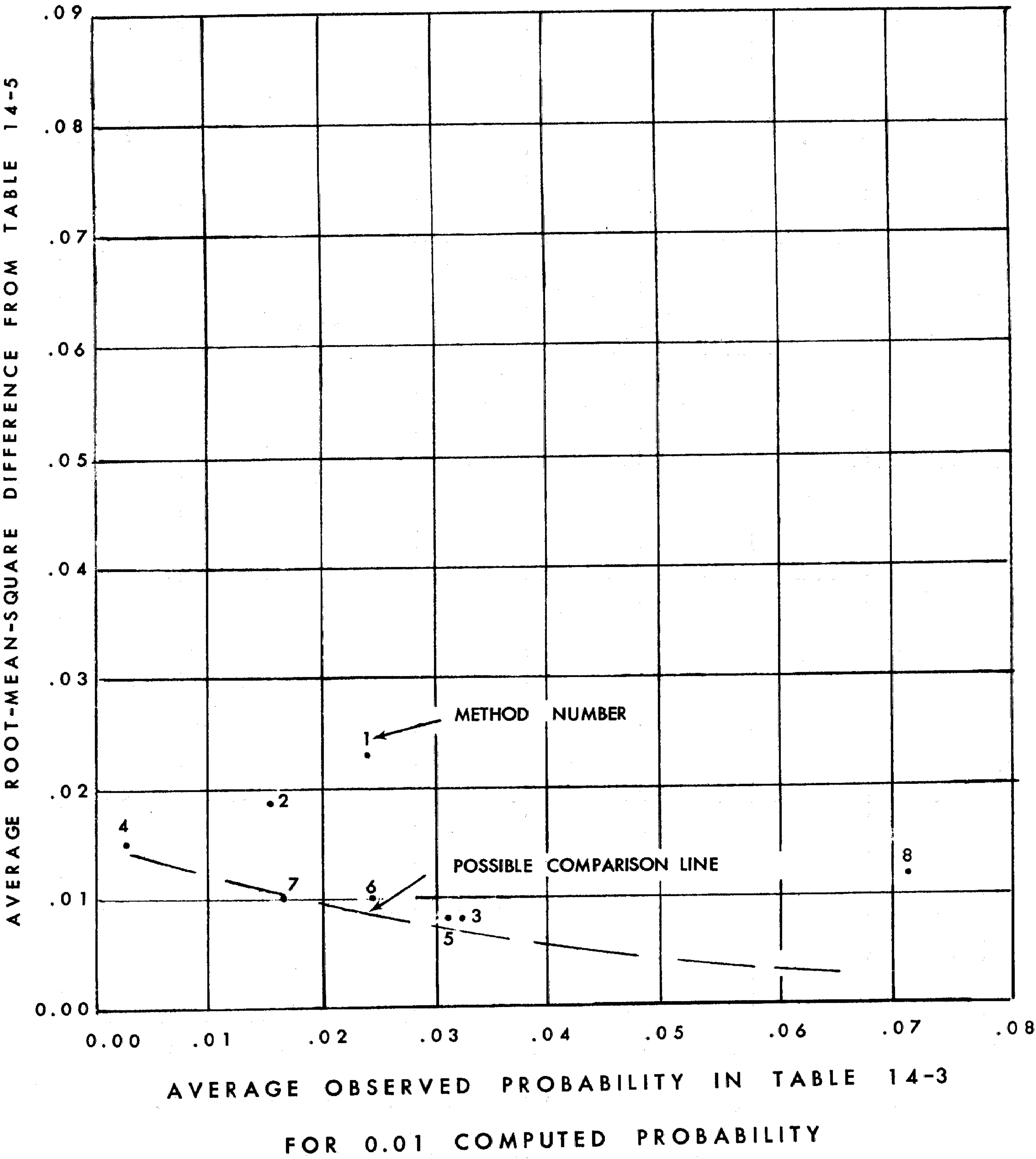


Table 14-1

Numbers of Verification Stations by Zones and Area Size

USGS	Drainage area category (sq. mi.)				Total
ZONE	0-25	25-200	200-1000	1000+	
1	4	8	10	5	27
2	2	5	12	5	24
3	5	3	16	1	25
4	1	6	8	0	15
5	3	2	14	1	20
6	4	3	13	4	24
7	5	2	12	2	21
8	8	2	11	2	23
9	1	7	8	2	18
10	0	8	4	0	12
11	2	5	6	0	13
12	0	5	9	3	17
13	0	2	10	5	17
14	0	6	8	1	15
15	2	1	0	0	3
16	12	1	0	0	13
*	4	7	1	1	13
Total	53	73	142	32	300

*Zero-flow stations (zones 8, 10 & 11 only)

Table 14-2

NUMBER OF STATIONS WHERE ONE OR MORE OBSERVED FLOOD EVENTS EXCEEDS THE 1000-YR FLOW COMPUTED FROM COMPLETE RECORD

ZONE	STATION-YEARS OF RECORD	METHOD 1	2	3	4	5	6	7	8
1	1414	0	1	8	0	10	7	2	26
2	1074	0	3	9	0	10	7	1	19
3	1223	1	3	7	0	9	8	4	22
4	703	1	2	3	0	3	3	2	12
5	990	2	1	7	0	4	4	0	19
6	1124	0	2	4	0	4	4	1	18
7	852	1	2	5	1	3	4	3	17
8	969	1	1	10	0	3	3	1	19
9	920	3	0	4	0	3	3	1	16
10	636	1	0	2	0	1	1	0	10
11	594	1	1	6	0	4	4	0	11
12	777	0	2	2	0	2	2	2	9
13	911	1	0	1	0	4	2	2	14
14	761	0	0	3	0	4	1	1	15
15	120	0	0	0	0	0	0	0	2
16	637	1	0	4	0	4	3	0	12
*	495	1	0	2	0	0	0	0	12
TOTAL	14,200	14	18	77	1	68	56	20	253

Based on the 14,200 station-years of record, it might be expected that about 14 observed events would exceed the true 1000-year magnitudes.

*Zero-flow stations

Table 14-3

STANDARD DEVIATION COMPARISONS

AVERAGE FOR ZONES 1 TO 16

COMPUTED PROBABILITY	METHOD 1	2	3	4	5	6	7	8
				AVERAGE OBSERVED PROBABILITIES				
.001	.0105	.0041	.0109	.0001	.0110	.0092	.0045	.0009
.01	.0232	.0153	.0315	.0023	.0309	.0244	.0170	.0015
.1	.1088	.1007	.1219	.0707	.1152	.1047	.1020	.0029
.5	.5090	.5149	.4576	.6152	.4713	.4950	.5108	.0037
	STANDARD DEVIATION OF OBSERVED PROBABILITIES FOR SPECIFIED COMPUTED PROBABILITIES							
.001	.0290	.0134	.0244	.0025	.0239	.0218	.0150	.0222
.01	.0430	.029	.045	.010	.043	.039	.032	.035
.1	.086	.084	.089	.074	.089	.084	.084	.067
.5	.132	.131	.142	.133	.133	.141	.130	.123

Note: Averages and standard deviations are of observed frequencies in the reserved portion of each record corresponding to computed mangitudes based on half records. Low standard deviations in relation to averages indicate more reliable estimates.

Table 14-4

Evaluation of Alternative Methods

Accuracy Tests b and c, Average Values, All Stations

Test b--Root mean square difference between plotting position and computed probability in other half of record.

	Method							
	1	2	3	4	5	6	7	8
Maximum	.062	.060	.067	.056	.070	.069	.061	.061
Decile	.084	.080	.097	.063	.098	.094	.081	.082
Median	.254	.105	.657	.193	.518	.295	.120	.727

Test c--Root mean square difference bewteen 1.0 and ratio of computed probability of flow in opposite half of record to plotting position. A zero value would indicate a perfect forecast.

	Method							
	1	2	3	4	5	6	7	8
Maximum	.53	.51	.56	.45	.56	.56	.51	.59
Decile	.37	.34	.38	.27	.37	.37	.34	.40
Median	.40	.12	.65	.19	.59	.44	.14	.52

Table 14-5

Evaluation of Alternative Methods

Consistency Tests a and b, Average Values, All Stations

Test a--Root mean square difference between computed probabilities from the two record halves for full record extreme, largest, upper decile and median events. A zero value would indicate perfect consistency.

Event	Method 1	2	3	4	5	6	7	8
Extreme	.003	.006	.001	.010	.001	.002	.003	.002
Maximum	.023	.019	.008	.016	.008	.010	.010	.012
Upper Decile	.072	.047	.043	.025	.037	.033	.025	.048
Median	.119	.076	.072	.047	.049	.045	.041	.131

Test b--Root mean square value of (1.0 minus the ratio of the smaller to the larger computed probabilities from the two record halves) for full record extreme, largest, upper decile and median events. A zero value would indicate perfect consistency.

Event	Method 1	2	3	4	5	6	7	8
Extreme	.87	.54	.46	.26	.39	.35	.29	.75
Maximum	.74	.45	.41	.21	.34	.30	.24	.72
Upper Decile	.50	.32	.31	.16	.24	.21	.17	.58
Median	.21	.14	.12	.10	.08	.08	.07	.24

Table 14-6
Evaluation of Outlier Techniques
Average Values, All Stations

Method

Accuracy Test b

Outlier Technique	1	2	3	4	5	6	7
a	.061	.062	.071	.057	.074	.073	.062
b	.056	.055	.060	.053	.063	.062	.055
c	.052	.050	.054	.048	.057	.055	.051
d	.047	.045	.048	.044	.051	.050	.045

Accuracy Test c

Outlier Technique	1	2	3	4	5	6	7
a	.53	.55	.57	.47	.58	.58	.54
b	.57	.59	.59	.49	.62	.60	.58
c	.58	.61	.60	.52	.64	.63	.60
d	.65	.65	.64	.38	.68	.65	.64

Consistency Test a

Outlier Technique	1	2	3	4	5	6	7
a	.002	.005	.001	.009	.000	.002	.002
b	.002	.004	.001	.008	.000	.002	.002
c	.003	.003	.000	.007	.000	.002	.002
d	.003	.003	.000	.007	.000	.002	.001

Consistency Test b

Outlier Techniques	1	2	3	4	5	6	7
a	.87	.56	.46	.27	.39	.36	.30
b	.86	.56	.45	.28	.38	.35	.30
c	.85	.56	.45	.29	.38	.35	.30
d	.88	.59	.45	.31	.38	.35	.32

A zero value would indicate perfect consistency.

Method 8 includes its unique technique for outliers and was, therefore, not included in these tests.

Table 14-7

Evaluation of Zero Flow Techniques

Average Values, All Stations

Accuracy Test b

Technique	Method 1	2	3	4	5	6	7
a	.057	.057	.059	.057	.062	.055	.059
b	.064	.060	.070	.057	.068	.061	.061

Accuracy Test c

Technique	Method 1	2	3	4	5	6	7
a	.46	.32	.59	.32	.40	.40	.32
b	.51	.30	.59	.30	.40	.41	.31

Consistency Test a

Technique	Method 1	2	3	4	5	6	7
a	.007	.012	.000	.014	.001	.000	.006
b	.007	.008	.000	.012	.000	.001	.004

Consistency Test b

Technique	Method 1	2	3	4	5	6	7
a	.89	.43	.44	.21	.39	.34	.24
b	.86	.43	.44	.19	.40	.38	.23

Method 8 was not tested because logarithms are not used in its fitting computations and therefore zero flows are not a problem.

Table 14-8

Summary of Partial-Duration Ratios

Zone	Partial-duration frequencies for annual-event frequencies of						
	.1	.2	.3	.4	.5	.6	.7
1 (21 sta)	.094	.203	.328	.475	.641	.844	1.10
2 (17 sta)	.093	.209	.353	.517	.759	1.001	1.30
3 (19 sta)	.094	.206	.368	.507	.664	.862	1.18
4 (8 sta)	.095	.218	.341	.535	.702	.903	1.21
5 (17 sta)	.093	.213	.355	.510	.702	.928	1.34
6 (16 sta)	.134	.267	.393	.575	.774	1.008	1.33
7 (9 sta)	.099	.248	.412	.598	.826	1.077	1.42
8 (12 sta)	.082	.211	.343	.525	.803	1.083	1.52
9 (15 sta)	.106	.234	.385	.553	.765	.982	1.26
10 (12 sta)	.108	.248	.410	.588	.776	1.022	1.34
11 (12 sta)	.094	.230	.389	.577	.836	1.138	1.50
12 (12 sta)	.103	.228	.352	.500	.710	.943	1.21
13 (16 sta)	.095	.224	.372	.562	.768	.986	1.30
14 (14 sta)	.100	.226	.371	.532	.709	.929	1.22
15 (3 sta)	.099	.194	.301	.410	.609	.845	1.05
16 (13 sta)	.106	.232	.355	.522	.696	.912	1.27
Average	.099	.243	.366	.532	.733	.964	1.28
Langbein	.105	.223	.356	.510	.693	.917	1.20

Note: Data limited to 226 stations originally selected for the study.

TABLE 14-9

ADJUSTMEMT RATIOS FOR 10-YEAR FLOOD

SAMPLE SIZE	ZONE 1		27 STATIONS			AVG 1/2 RECORD = 26 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	.54	.38	.76	.29	.82	.57	.28	-1.85
10-YR	.75	.45	1.02	-.27	.95	.37	.34	4.56
1/2-REC	1.21	1.11	2.21	-1.04	2.01	1.01	1.03	4.49
	ZONE 2		24 STATIONS			AVG 1/2 RECORD = 22 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	.48	.42	1.06	.64	1.03	.93	.41	-1.85
10-YR	1.01	.94	1.91	.68	1.60	1.31	.80	5.70
1/2-REC	1.33	1.33	2.76	-1.58	1.90	.49	.54	7.14
	ZONE 3		25 STATIONS			AVG 1/2 RECORD = 24 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.41	1.32	1.92	1.02	1.95	1.79	1.40	-1.85
10-YR	1.41	.81	1.80	.00	1.87	.96	1.01	5.39
1/2-REC	.98	.14	1.65	-1.88	1.17	.21	.39	4.80
	ZONE 4		15 STATIONS			AVG 1/2 RECORD = 23 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.05	.94	1.20	.85	1.29	1.15	.94	-1.85
10-YR	-.52	-.50	.12	-.85	-.01	-.54	-.45	3.68
1/2-REC	.45	.02	1.63	-3.07	1.63	.46	.25	5.57
	ZONE 5		20 STATIONS			AVG 1/2 RECORD = 25 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	.55	.35	1.03	.15	.98	.88	.47	-1.85
10-YR	.40	-.03	1.40	-.96	.61	.42	.19	7.37
1/2-REC	.81	-.40	2.91	-3.61	1.42	.99	.67	6.23
	ZONE 6		24 STATIONS			AVG 1/2 RECORD = 23 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	.80	.36	1.19	.15	1.11	.95	.45	-1.85
10-YR	1.43	.18	2.26	-.98	1.78	.96	.33	5.64
1/2-REC	1.08	-.45	2.94	-3.93	1.94	.07	-.04	6.14
	ZONE 7		21 STATIONS			AVG 1/2 RECORD = 20 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.15	1.19	1.69	1.29	1.62	1.59	1.29	-1.85
10-YR	1.58	1.36	2.34	.12	1.99	1.62	1.57	5.78
1/2-REC	1.97	1.00	2.45	-.74	2.07	.92	1.17	7.11
	ZONE 8		23 STATIONS			AVG 1/2 RECORD = 21 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	.89	.79	1.71	.79	1.41	1.36	.79	-1.85
10-YR	-.66	-1.02	.29	-2.04	-.35	-.43	-1.02	4.52
1/2-REC	-.13	-.87	2.28	-3.08	.74	.66	-.87	7.88

TABLE 14-9 CONTINUED

	ZONE 9		18 STATIONS			AVG 1/2 RECORD = 25 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.38	1.02	2.05	.96	1.96	1.78	1.10	-1.85
10-YR	1.95	1.54	2.54	.75	2.49	2.22	1.69	5.76
1/2-REC	.45	-.36	.97	-3.36	.45	-.07	-.27	4.07
	ZONE 10		12 STATIONS			AVG 1/2 RECORD = 26 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	-.79	-.80	-.41	-.83	-.43	-.43	-.77	-1.85
10-YR	-.03	-.42	.90	-1.16	.71	.35	-.22	4.24
1/2-REC	.08	-1.27	1.24	-5.10	.58	-.27	-1.27	2.97
	ZONE 11		13 STATIONS			AVG 1/2 RECORD = 23 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.29	1.21	1.89	1.20	1.93	1.75	1.11	-1.85
10-YR	1.11	1.03	2.21	.04	1.87	1.25	1.03	6.78
1/2-REC	.04	-.23	1.99	-2.93	1.20	1.20	-.23	5.32
	ZONE 12		17 STATIONS			AVG 1/2 RECORD = 23 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.34	.73	1.34	.57	1.51	1.03	.80	-1.85
10-YR	.79	.41	.86	-.45	.92	-.44	.57	4.06
1/2-REC	.19	-.31	.54	-2.94	.92	-.35	-.19	2.81
	ZONE 13		17 STATIONS			AVG 1/2 RECORD = 26 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.27	1.16	1.65	.96	1.77	1.52	1.19	-1.85
10-YR	.26	.22	.88	-.83	.67	.42	.38	4.60
1/2-REC	-.31	-1.52	.21	-4.89	.17	-.97	-1.12	2.88
	ZONE 14		15 STATIONS			AVG 1/2 RECORD = 25 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.72	1.65	2.12	1.61	2.19	2.00	1.65	-1.85
10-YR	2.60	2.50	3.17	1.88	2.82	1.87	2.56	6.80
1/2-REC	.51	.61	1.83	-1.47	1.30	.29	.75	5.22
	ZONE 15		3 STATIONS			AVG 1/2 RECORD = 20 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	2.47	2.47	2.74	2.55	2.66	2.28	2.28	-1.85
10-YR	1.27	1.27	1.58	1.27	1.58	1.58	1.27	2.65
1/2-REC	3.29	3.29	3.29	2.79	3.29	1.90	3.29	6.33
	ZONE 16		13 STATIONS			AVG 1/2 RECORD = 24 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	.69	.75	1.03	.66	1.09	1.05	.75	-1.85
10-YR	.58	.42	.83	-.21	.76	.07	.42	4.24
1/2-REC	1.41	.07	1.68	-3.43	1.25	.64	.07	5.29
	ALL ZONES		287 STATIONS			AVG 1/2 RECORD = 23 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	.94	.79	1.38	.71	1.37	1.21	.81	-1.85
10-YR	.87	.52	1.52	-.29	1.26	.72	.60	5.27
1/2-REC	.77	.04	1.93	-2.66	1.34	.40	.17	5.36

Values shown are ratios by which the theoretical adjustment for Gaussian-distribution samples must be multiplied in order to convert from the computed 0.1 probability to average observed probabilities in the reserved data. See note table 14-11.

TABLE 14-10

ADJUSTMENT RATIOS FOR 100-YEAR FLOOD

SAMPLE SIZE	ZONE 1		27 STATIONS			AVG 1/2 RECORD = 26 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.35	1.11	1.27	.39	1.61	1.12	.88	-.25
10-YR	1.50	1.10	2.05	-.25	2.42	1.73	.73	3.42
1/2-REC	2.83	2.84	3.90	-1.06	4.89	3.67	1.66	5.28
	ZONE 2		24 STATIONS			AVG 1/2 RECORD = 22 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	.91	.79	1.05	.31	1.27	1.13	.63	-.25
10-YR	1.44	1.40	2.48	.63	2.41	2.07	1.37	5.40
1/2-REC	1.00	1.08	3.69	-.82	2.97	2.46	.14	7.16
	ZONE 3		25 STATIONS			AVG 1/2 RECORD = 24 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.80	1.18	1.76	.41	2.05	1.86	1.29	-.25
10-YR	2.42	1.15	2.43	-.04	2.84	1.62	1.32	4.79
1/2-REC	2.90	1.41	3.36	-1.12	3.71	2.76	2.30	5.53
	ZONE 4		15 STATIONS			AVG 1/2 RECORD = 23 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.67	1.48	1.45	.59	2.27	2.02	1.64	-.25
10-YR	.67	.35	.56	-.48	1.07	.46	.42	1.50
1/2-REC	1.86	.48	1.54	-1.15	2.83	.88	1.03	3.81
	ZONE 5		20 STATIONS			AVG 1/2 RECORD = 25 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.03	.64	1.37	.24	1.19	1.12	.82	-.25
10-YR	1.22	.57	1.42	-.29	1.27	1.09	.80	5.65
1/2-REC	2.97	.21	4.38	-1.24	2.97	2.39	1.68	7.25
	ZONE 6		24 STATIONS			AVG 1/2 RECORD = 23 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.15	.67	1.02	.04	1.17	.88	.76	-.25
10-YR	2.30	.55	1.67	-.27	1.78	1.10	.66	4.43
1/2-REC	1.20	-.23	3.22	-1.24	2.45	.79	.46	5.09
	ZONE 7		21 STATIONS			AVG 1/2 RECORD = 20 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.04	1.07	2.23	.28	2.20	2.16	1.20	-.25
10-YR	1.18	1.09	2.66	-.19	2.54	2.20	1.53	5.40
1/2-REC	3.10	.47	3.92	-.80	2.99	2.29	1.74	8.33
	ZONE 8		23 STATIONS			AVG 1/2 RECORD = 21 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	.57	.27	2.08	.01	1.66	1.52	.27	-.25
10-YR	1.30	.14	1.59	-.35	1.15	.93	.14	4.17
1/2-REC	.82	-.32	4.36	-1.13	2.16	2.16	-.32	8.49

TABLE 14-10 CONTINUED

	ZONE 9		18 STATIONS			AVG 1/2 RECORD = 25 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.07	1.33	1.90	.72	2.11	2.11	1.50	-.25
10-YR	2.45	2.23	3.21	.90	3.75	3.55	2.57	4.39
1/2-REC	1.07	.39	2.90	-1.72	3.78	2.38	.66	4.49
	ZONE 10		12 STATIONS			AVG 1/2 RECORD = 26 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	-.10	-.10	.27	-.25	.29	.29	-.06	-.25
10-YR	.21	-.15	.96	-.59	1.06	.75	.15	2.55
1/2-REC	3.29	-.27	1.63	-1.79	2.42	1.32	-.27	4.40
	ZONE 11		13 STATIONS			AVG 1/2 RECORD = 23 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	.68	.70	1.79	.11	1.58	1.54	.66	-.25
10-YR	2.41	1.51	4.14	.17	3.76	3.43	1.28	6.64
1/2-REC	.30	.79	5.40	-1.08	3.05	2.43	.50	9.77
	ZONE 12		17 STATIONS			AVG 1/2 RECORD = 23 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.81	1.10	1.16	.44	1.56	1.19	1.19	-.25
10-YR	1.99	1.93	1.55	.13	2.27	1.04	2.11	2.60
1/2-REC	3.77	1.65	2.12	-1.33	4.39	2.57	1.86	1.82
	ZONE 13		17 STATIONS			AVG 1/2 RECORD = 26 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.63	.87	1.12	.50	1.63	1.26	1.04	-.25
10-YR	.58	.37	1.27	-.28	1.41	1.25	.60	3.28
1/2-REC	1.01	-.07	2.20	-1.81	2.57	1.61	.81	2.69
	ZONE 14		15 STATIONS			AVG 1/2 RECORD = 25 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.54	1.44	1.79	.65	2.43	2.21	1.44	-.25
10-YR	2.92	2.22	2.58	.23	3.53	1.98	2.32	5.16
1/2-REC	2.11	2.80	3.76	-1.52	4.40	3.10	2.80	5.37
	ZONE 15		3 STATIONS			AVG 1/2 RECORD = 20 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	2.09	2.24	2.24	1.24	2.76	1.98	1.50	-.25
10-YR	.26	.26	.26	-.59	1.84	1.84	.26	1.72
1/2-REC	1.80	1.80	.93	-1.31	4.37	3.16	.93	.93
	ZONE 16		13 STATIONS			AVG 1/2 RECORD = 24 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	.61	.55	.90	.18	1.30	1.22	.62	-.25
10-YR	1.87	1.23	1.63	-.59	1.83	.99	1.33	3.64
1/2-REC	4.21	1.17	3.96	-1.27	4.41	2.90	2.13	4.46
	ALL ZONES		287 STATIONS			AVG 1/2 RECORD = 23 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.16	.90	1.45	.32	1.65	1.45	.94	-.25
10-YR	1.64	1.03	2.01	-.07	2.20	1.62	1.12	4.25
1/2-REC	2.12	.87	3.40	-1.23	3.35	2.30	1.14	5.66

Values shown are ratios by which the theoretical adjustment for Gaussian-distribution samples must be multiplied in order to convert from the computed 0.01 probability to average observed probabilities in the reserved data. See note table 14-11.

TABLE 14-11

ADJUSTMENT RATIOS FOR 1000-YEAR FLOOD

SAMPLE SIZE	ZONE 1		27 STATIONS			AVG 1/2 RECORD = 26 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	2.03	1.10	1.19	.21	2.12	1.44	.85	-.04
10-YR	2.30	.88	2.21	-.14	2.98	1.87	.52	4.06
1/2-REC	5.01	4.13	6.94	-.56	10.11	8.16	1.66	8.54
	ZONE 2		24 STATIONS			AVG 1/2 RECORD = 22 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.31	.83	1.18	.15	1.57	1.35	.68	-.04
10-YR	1.98	2.85	3.85	.64	4.45	3.66	2.07	7.41
1/2-REC	1.93	2.11	4.47	-.45	3.56	3.56	1.58	8.81
	ZONE 3		25 STATIONS			AVG 1/2 RECORD = 24 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	2.42	1.22	2.18	-.01	2.54	2.08	1.24	-.04
10-YR	6.06	2.20	3.06	-.14	3.89	1.82	2.20	7.11
1/2-REC	7.41	2.44	6.77	-.51	7.06	4.82	2.77	11.16
	ZONE 4		15 STATIONS			AVG 1/2 RECORD = 23 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.88	1.50	1.46	.30	2.48	2.05	1.63	-.04
10-YR	1.24	.54	.47	-.14	1.13	.36	.71	1.33
1/2-REC	2.86	.80	2.11	-.48	3.60	3.60	2.40	2.81
	ZONE 5		20 STATIONS			AVG 1/2 RECORD = 25 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.84	.94	1.36	.49	1.92	1.45	1.32	-.04
10-YR	2.75	.56	2.90	-.14	2.43	2.00	.91	6.02
1/2-REC	5.51	1.39	5.76	-.52	5.89	5.30	3.22	11.70
	ZONE 6		24 STATIONS			AVG 1/2 RECORD = 23 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.91	.61	1.08	.07	1.54	1.13	.79	-.04
10-YR	3.99	.57	1.73	-.06	2.33	1.57	1.12	4.53
1/2-REC	2.88	1.38	2.47	-.48	2.06	1.63	1.24	8.92
	ZONE 7		21 STATIONS			AVG 1/2 RECORD = 20 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.19	.82	1.91	.19	2.18	1.89	1.40	-.04
10-YR	2.33	.96	3.58	.13	3.25	2.15	1.53	6.52
1/2-REC	5.99	1.48	5.36	.16	3.90	3.90	2.34	12.51
	ZONE 8		23 STATIONS			AVG 1/2 RECORD = 21 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	.83	.09	1.28	-.01	.83	.83	.14	-.04
10-YR	2.79	.42	2.68	-.14	1.78	1.78	.42	5.90
1/2-REC	2.70	.84	7.62	-.41	3.54	3.54	1.32	13.61
	ZONE 9		18 STATIONS			AVG 1/2 RECORD = 25 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	.90	1.30	1.37	.49	2.33	2.33	1.55	-.04
10-YR	3.61	3.59	3.22	.42	5.85	5.85	3.90	6.24
1/2-REC	3.59	.59	3.97	-.53	2.68	1.04	1.07	6.92

TABLE 14-11 CONTINUED

	ZONE 10		12 STATIONS			AVG 1/2 RECORD = 26 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	.02	-.04	.25	-.04	.22	.22	-.04	-.04
10-YR	.44	-.14	.70	-.14	.67	.43	-.14	3.79
1/2-REC	7.21	.27	3.04	-.56	1.95	1.95	.27	4.50

	ZONE 11		13 STATIONS			AVG 1/2 RECORD = 23 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.13	1.01	2.15	.20	2.13	1.78	.94	-.04
10-YR	4.31	2.44	5.95	.72	5.06	3.58	1.90	10.41
1/2-REC	1.74	.91	6.38	-.46	5.01	4.24	.91	15.65

	ZONE 12		17 STATIONS			AVG 1/2 RECORD = 23 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	2.84	1.22	1.31	.45	2.03	1.51	1.27	-.04
10-YR	4.30	2.17	2.52	.10	4.27	1.40	2.17	3.37
1/2-REC	8.58	.75	.75	-.46	2.20	1.34	.75	4.59

	ZONE 13		17 STATIONS			AVG 1/2 RECORD = 26 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.89	1.21	1.11	.32	1.92	1.79	1.21	-.04
10-YR	1.27	.36	1.39	-.14	1.77	1.77	.53	3.56
1/2-REC	4.01	-.57	2.83	-.57	3.65	2.43	.55	4.96

	ZONE 14		15 STATIONS			AVG 1/2 RECORD = 25 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.91	1.45	1.56	.47	2.66	2.03	1.45	-.04
10-YR	5.41	2.35	2.81	-.14	4.63	2.17	2.35	5.56
1/2-REC	3.45	1.04	5.12	-.53	9.90	6.99	1.04	6.69

	ZONE 15		3 STATIONS			AVG 1/2 RECORD = 20 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	2.67	3.00	2.54	-.04	3.51	1.25	1.77	-.04
10-YR	-.14	-.14	-.14	-.14	1.87	1.87	-.14	-.14
1/2-REC	2.17	2.17	-.38	-.38	6.15	6.15	-.38	-.38

	ZONE 16		13 STATIONS			AVG 1/2 RECORD = 24 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	.69	.62	1.15	-.04	1.40	1.18	.69	-.04
10-YR	4.02	1.56	3.05	-.14	3.90	1.97	2.01	4.46
1/2-REC	8.74	2.37	7.24	-.51	8.30	6.21	3.76	7.24

	ALL ZONES		287 STATIONS			AVG 1/2 RECORD = 23 YRS		
METHOD	1	2	3	4	5	6	7	8
5-YR	1.60	.95	1.40	.21	1.89	1.54	1.01	-.04
10-YR	3.13	1.40	2.66	.04	3.22	2.19	1.45	5.36
1/2-REC	4.66	1.49	4.81	-.45	4.99	4.02	1.68	8.80

Values shown are ratios by which the theoretical adjustment for Gaussian-distribution samples must be multiplied in order to convert from the computed 0.001 probability to average observed probabilities in the reserved data.

Table 14-11 CONTINUED

Values in table 14-11 are obtained as follows:

a. Compute the magnitude corresponding to a given exceedance probability for the best-fit function.

b. Count proportion of values in remainder of record that exceed this magnitude.

c. Subtract the specified probability from b.

d. Compute the Gaussian deviate that would correspond to the specified probability.

e. Compute the expected probability for the given sample size (record length used) and the Gaussian deviate determined in d.

f. Subtract the specified probability from e.

g. Divide f by c.

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